MRI turbulence in accretion discs: angular momentum transport in the low Prandtl number limit

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The mechanisms and rate of angular momentum transport in astrophysical discs is a central problem in accretion theory. Currently the most widely studied mechanism for this transport is the turbulence induced by the magnetorotational instability (MRI). However previous studies have proven that this transport decreases with a decreasing ratio of fluid viscosity to fluid resistivity, also called Prandtl number $Pm=\nu/\eta$. This questions the role of MRI turbulence in discs with very low Prandtl number such as protoplanetary discs. In this context, we study the rate of angular momentum transport at low Prandtl number by the means of local simulations of MRI-driven turbulence at very high resolution. In this talk, I will present results showing a convergence of transport rate at low $Pm$ for different magnetic field configurations and discuss the relevance of implicit Large Eddy Simulations (LES) for such a study.

Subject : oral
Topics : Astrophysics
MRI TURBULENCE IN ACCRETION DISCS: Angular momentum transport in the low Pm limit

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ACCRETION PROCESS

\[ \nu = \alpha c_s H \]
\[ \alpha \sim 10^{-3} - 10^{-1} \]
MAGNETO-ROTATIONAL INSTABILITY

- Linear mechanism
- Turbulence
- Low Pm limit: no transport?

\[ \beta = 10^2 \]
\[ \beta = 10^3 \]
\[ \beta = 10^4 \]

\[ R_e = 400 \]
\[ R_e = 800 \]
\[ R_e = 1600 \]
\[ R_e = 3200 \]
\[ R_e = 6400 \]
\[ R_e = 20000 \text{ & } \beta = 10^3 \]

(Lesur & Longaretti 2010)
SHEARING BOX SIMULATIONS

- Shearing box boundary conditions
- Ramses code
- Isothermal disc \( P = c_0^2 \rho \)
- Unstratified disc

\[
\begin{align*}
\frac{\partial_t \rho}{\rho v} + \nabla \cdot (\rho v) &= 0 \\
\frac{\partial (\rho v)}{\partial t} + \nabla \cdot (\rho vv - BB) &= 2 \rho \Omega_0 x i - 2 \rho \Omega_0 \times v + \nabla \cdot T - \nabla P_{tot} \\
\partial_t B &= \nabla \times (v \times B - \eta \nabla \times B) \\
T_{ij} &= \nu \rho (\partial_j v_i + \partial_i v_j - \frac{2}{3} \delta_{ij} \nabla \cdot v)
\end{align*}
\]
NON IDEAL SIMULATIONS

- Viscosity and resistivity coefficient
- High resolution is needed! (Fromang et al. 2007)

\[ Re = \frac{\Omega L_z^2}{\nu} \]
\[ Rm = \frac{\Omega L_z^2}{\eta} \]
\[ Pm = \frac{Rm}{Re} \]

<table>
<thead>
<tr>
<th>Model</th>
<th>Resolution</th>
<th>Re</th>
<th>Rm</th>
<th>Pm</th>
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<tr>
<td>Y-C-Re650</td>
<td>(64, 128, 64)</td>
<td>650</td>
<td>2600</td>
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<td>8000</td>
<td>400</td>
<td>0.05</td>
</tr>
</tbody>
</table>
THE ‘GRAND CHALLENGE’ SIMULATION

- Toroidal magnetic field $\beta \sim 100$
- Two steps:

  1. Ideal step:

     $Re = \infty \quad Rm = \infty$

  2. Viscous and resistive step:

     $Pm = 0.03 \quad Re = 85000 \quad Rm = 2600$

1.06 time-steps
1.07 CPU hours (Blue Gene)
$2^{15}$ cores
131,072 threads
THE ‘GRAND CHALLENGE’ SIMULATION
$\alpha_{\text{mean}} = 0.0179 \pm 3.16 \times 10^{-3}$

$\alpha_{\text{Rey}} = <\rho \delta v_r \delta u_\phi>$

$\alpha_{\text{Max}} = -<B_r B_\phi>$

$E_M(k) = <B_k^2(k)/2>$

$E_K(k) = <\rho v_k^2(k)/2>$
CONVERGENCE AT LOW PM
$E_M(k) = \langle B_k^2(k)/2 \rangle$

$E_K(k) = \langle \rho v_k^2(k)/2 \rangle$

$E_k \propto k^{-3/2}$
\[ \text{Large Eddy Simulations (ILES)} \]

\[ Rm = 2600 - 400 \]

\[ Re = \infty \]
SUMMARY & DISCUSSION