Causality and stability of cosmic jets
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Cosmic jets are able to transport energy over vast spaces which exceed by up to a billion times the size of their central engines. We propose that the reason behind this remarkable property is the loss of causal connectivity across these jets, caused by their rapid expansion in response to fast decline of external pressure with the distance from the "jet engine". In order to verify this claim, we have carried out numerical simulations of moderately magnetized and moderately relativistic jets that expand and accelerate due to the decrease of ambient pressure. The results give strong support to our hypothesis and provide valuable insights on the mechanism of jet disruption. In particular, we find that the z-pinched inner cores of magnetic jets expand slower than their envelopes and become susceptible to instabilities even when the jet is stable on the global scale. This may result in local dissipation and emission without total disintegration of the flow. Cosmic jets may become globally unstable when they enter flat sections of external atmospheres. We propose that the Fanaroff-Riley morphological division of extragalactic radio sources into two classes is related to this issue. In particular, we argue that the low power FR-I jets become re-confined, causally connected and globally unstable on the scale of galactic X-ray coronas, whereas more powerful FR-II jets re-confine much further out, already on the scale of radio lobes, and remain largely intact until they terminate at hot spots.

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A biased introduction

(AGN) Jets cover $10^5$-$10^7$ of initial radii without disruption

Except when they don’t:

Optical synchrotron suggests continuous particle re-acceleration thus dissipation (e.g. Meisenheimer 2003)

How can we avoid disrupting fluid instabilities?

What is the deciding factor between the FR1 and FR2 division?

How can we have dissipation without disruption?
Dangerous jet instabilities

Current driven instabilities (CDI): Pinch, Kink and higher orders (Bateman (1978), Begelman (1998), Appl et al. (2000), Baty (2005), Lery et al. (2000); Narayan et al. (2009); Moll et al (2011); Mizuno et al (2011,12,14); O’Neill et al (2012); Mignone et al. (2013); Anjiri+ (2014); ...)

Can be stabilized with: Relativistic bulk motion, shear, being force-free, electric fields, poloidal fields, jet expansion

Kelvin Helmholtz type instabilities (KHI):
(Turland & Scheuer (1976); Ferrari+ (1978); Hardee (2004); Perucho et al (2004,07); Bodo et al (2006); Rossi et al (2008); ...)

Can be stabilized with: Relativistic bulk motion, Thick shear layer, poloidal fields, jet expansion
To disrupt the jet we need global instabilities.

For global instability, transverse causal connection is required, sufficient condition:

\[ \theta_M > \theta_v \]

Flow or opening angle: \( \theta_v \approx r/z \)

Mach angle of fastest wave: \( \theta_M \)

“The jet is causally disconnected when the Mach cone does not point to the axis”

Hot hydro jet: \( \theta_M = 1/\Gamma \); ... \( \Gamma \theta > 0.7 \)

RMHD case: \( \theta_M \approx \sqrt{\frac{\mu}{\Gamma^3}} \)

\( \mu \) : Total energy flux per rest mass

MOJAVE AGN-Survey: \( \Gamma \theta \approx 0.2 \) (Clausen-Brown et al. (2014))

Also holds for efficient RMHD acc.Komissarov et al. (2009)
Jet-galaxy connection

- Much interest in feedback from jet to galaxy
- Jet “feels” galaxy via its decreasing pressure profile (also: self-similar feedback cycle, Falle 1991)

Central pressure:
- $M_\bullet = 10^9 M_\odot$
- $p(r_g) = p(1.5 \times 10^{14} \text{ cm}) \simeq (10^7 / \alpha) \text{ dyn cm}^{-2}$

Lobe pressure:
- $p(100 \text{ kpc}) \simeq 10^{-11} \text{ dyn cm}^{-2}$

Gives single powerlaw index of $\kappa \approx 2$
- $p(r) \propto r^{-\kappa}$

Galactic pressure distribution:
Beta-law, (e.g. Mathews & Brighenti 2003)

\[ p = p_0 (1 + (z / z_0)^2)^{-\kappa / 2} \]

$z_0 = 1 \text{ kpc}; \kappa = 1.25 \pm 0.25; p_0 \approx 10^{-9} \text{ dyn cm}^{-2}$

**Figure 1.** Profiles of the hot gas pressure in M 87 host galaxy, as evaluated by Falle & Wilson (1985, dashed line), Owen et al. (1989, thin solid line) and in this paper (thick solid line). Circles indicate minimum pressure of the knots in the M 87 jet neglecting the relativistic correction (filled ones), and assuming the jet Doppler factor $\delta = 2.7$ (open ones). The circles disconnected from the others correspond to the HST-1 flaring region (the upstream edge of the HST-1 knot). In deprojecting distances between the knots and the active core, we assumed the jet viewing angle of $\theta = 20^\circ$. 

Stawarz+ 2006
Causally (dis-)connected jets

In pressure confined flows with \( p_{\text{ext}} \propto r^{-\kappa} \), we have for

- **Unmagnetized** flows, hot and cold: \( \theta_M \propto \theta_v z^{(2-\kappa)/2} \)
  
  => Free/conical expansion for \( \kappa > 2 \)

- Cold unmagnetized jets **reconfine** only for \( \kappa < 2 \)
  (Falle & Komissarov 1997)

- **Pointing dominated** jets expand freely if \( \kappa > 2 \)
  (Komissarov+ 2009; Lyubarsky 2009)

\( \kappa = 2 \) is a critical value for many types of flows
Causally (dis-)connected jets

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\( \rightarrow \) Free/conical expansion for \( \kappa > 2 \)

Opening up a confined GRB jet: Komissarov+ (2010), Tchekhovskoy+ (2010)
Strategy

- Study stability of expanding jets in atmospheres of various steepness $\kappa$
- Steady-state solutions are not readily available for linear analysis
- Non-linear phases are of particular interest as they relate to observations directly

Hence, numerical simulations are required.

The huge difference in length scales prohibits direct approach. …at least at high resolution...

Try a more creative way…
Approximate steady-state solutions

Investigate e.g. 2D stationary continuity equation:

\[ \partial_x \Gamma \rho v^x + \partial_y \Gamma \rho v^y = 0 \]

\( (v_x \sim c) \Rightarrow c \partial_x \Gamma \rho + \partial_y \Gamma \rho v^y = 0 \)

\( (x \sim ct; c \partial_x = \partial_t) \Rightarrow \partial_t \Gamma \rho + \partial_y \Gamma \rho v^y = 0 \)

Boundary condition: \( p(x,y_{\text{end}}) \Rightarrow p(ct,y_{\text{end}}) \)

Proceed in analogue fashion for remaining MHD equations

\[ \Rightarrow \text{Approximate 2D steady state solutions from 1D time-dependent simulations!} \]

Valid for supersonic flows with \( \Gamma \gg 1; v^x \gg v^y \); good for highly relativistic collimated jets.
Approximate steady-state solutions

Numerically this resembles a "Marching scheme".

Steady supersonic relativistic hydro jets where investigated with a marching scheme e.g. by Bowman (1994)

\[ \propto z^{-2} \]

=> Good agreement with our approximate solutions!

Pressure and temperature contours in the M=15 relativistic steady jets of Bowman (1994) –bottom and our approximate solution –top.
Adopt cylindrical "core-envelope" model of Komissarov (1999)

Force-free $b^\Phi \propto 1/r$ envelope and z-pinched core

Kink unstable (Begelman 1998, O'Neill+ 2012)

Powerlaw atmosphere with $p \propto z^{-k}$

Initial Lorentz factor $\Gamma_0=3$, average magnetization $\sigma=0.2$
Waves traveling back and forth the jet giving rise to oscillations of jet boundary

Slowly expanding dense core develops

For $\kappa \geq 2$ flow is conical and no waves traverse across the jet

Jet boundary indeed becomes disconnected for $\kappa \geq 2$
Simple MHD jet model

- Waves traveling back and forth the jet giving rise to oscillations of jet boundary
- Slowly expanding dense core develops
- For $\kappa \geq 2$ flow is conical and no waves traverse across the jet
- Jet boundary indeed becomes disconnected for $\kappa \geq 2$
Periodic box simulations of expanding jets

Evolve with ideal RMHD module of AMRVAC\(^1\)

Perturb with:

\[ v^r(r, \phi, z) = \frac{\Delta v}{N} \exp\left(-\frac{r}{r_m}\right) \sum_{n=1}^{N} \cos \phi \sin(2\pi n z/L_z), \]

\(n=4; \ \Delta v=0.01c\)

Typical domain size: \((L_x, L_y, L_z) = (192, 192, 64)r_j\)

AMR-grid follows expanding jet

Drive external gas to satisfy \(p_{ext} = p_0(t/t_0)^{-K}\)

Advect tracer sets jet boundary

Periodic boundary

\(\Gamma\)

AMRVAC

\(^1\)https://gitorious.org/amrvac

Thursday, 2 October 14
Periodic box simulations of expanding jets

1. jet accelerates
2. jet fragments
3. waves emitted into ambient medium

Instability develops in the jet core

y=0 slice for $\kappa=1$; $\Gamma; \kappa = 1; t = 0$

$\rho; \kappa = 1; t = 0$
Periodic box simulations of expanding jets

\( \kappa = 0.5 \)

Comparison of energetics from steady state models (dashed) with the full 3D case (solid).
Periodic box simulations of expanding jets

Comparison of energetics from steady state models (dashed) with the full 3D case (solid).

Ample magnetic dissipation but less than 1% loss of jet power!

Thursday, 2 October 14
Periodic box simulations of expanding jets

\[ \kappa = 1.5 \]

\( \Gamma; \kappa = 1.5; t = 1000 \)

\( \rho; \kappa = 1.5; t = 1000 \)

\( \Gamma; \kappa = 1.5; t = 2000 \)

\( \rho; \kappa = 1.5; t = 2000 \)
Periodic box simulations of expanding jets

\[ \kappa = 2 \]

\[ \Gamma; \kappa = 2; t = 3000 \]

\[ \rho; \kappa = 2; t = 3000 \]

Comparison of energetics from steady state models (dashed) with the full 3D case (solid).
Periodic box simulations of expanding jets

Jet disruption is connected to displacement of the average barycenter:

\[ \bar{r} = \frac{\left| \int Q r ds \right|}{\int Q ds} \]

\[ Q = \Gamma^2 (\rho c^2 + 4p) \tau \]

where we take the rel. inertia times jet tracer as test-function \( Q \).

Jet disrupts if \( \bar{r} \sim 1/2r_{\text{jet}} \)

Increasing \( \kappa \) delays jet disruption.
Constant atmosphere case \( \kappa=0 \) disrupts after \( t=100 \)
No disruption occurs for \( \kappa=2 \) up to time \( t=3000 \) – as expected
Next steps: Instability triggered by reconfinement?

Jump conditions of cold relativistic flow (Komissarov & Falle 1997):

\[ p_2 = \mu \rho_j u_j^2 \sin(\alpha)^2 \]

\( \alpha \): Shock obliqueness; \( \mu = 17/24 \)

Use small angle approximation:

\[ \sin^2(\alpha) \approx \left( \frac{r}{z} - \frac{dr}{dz} \right)^2 \]

Set shocked pressure \( p_2 \) equal to ambient, e.g.

\[ P_{\text{ext}} = p_0 (z/z_0)^{-\kappa} \]

\[ z_r \approx z_0 (\delta A)^{1/\delta} \]

\[ \delta = 1 - \kappa/2 \]

\[ A \approx \frac{1}{3} p_0^{-1/2} L_j^{1/2} z_0^{-1} \]

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Yields reconfinement scale:

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Repeat this calculation for realistic galactic pressure profile and jet powers

Galactic pressure distribution:
Beta-law, (e.g. Mathew&Brighenti 2003)

\[ p = p_0 (1 + (z/z_0)^2)^{-\kappa/2} \]

\( z_0 = 1 \text{kpc}; \kappa = 1.25 \pm 0.25; p_0 = 10^{-9} \text{dyn cm}^{-2} \)

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\[ p = p_0 \left( 1 + \left( \frac{z}{z_0} \right)^2 \right)^{-\kappa/2} \]

\( z_0=1 \) kpc; \( \kappa=1.25 \pm 0.25; \ p_0=10^{-9} \) dyn cm\(^{-2} \)

Yields reconfinement scale:

\[ z_r \simeq z_0 \left( \delta A \right)^{1/\delta} \]

\( \delta = 1 - \kappa/2 \)

\[ A \simeq \frac{1}{3} p_0^{-1/2} L_{j,44}^{1/2} z_0^{-1} \text{kpc} \]

Repeat this calculation for realistic galactic pressure profile and jet powers.

\[
\begin{align*}
\log_{10} p &\quad \text{Logarithmic pressure scale}\n0 &\quad \text{Minimum pressure value}\n10^{-3.2} &\quad \text{Maximum pressure value}\n\end{align*}
\]

\[
\begin{align*}
\log_{10} A &\quad \text{Logarithmic jet power scale}\n0 &\quad \text{Minimum jet power value}\n10^{-2} &\quad \text{Maximum jet power value}\n\end{align*}
\]
Next steps: Instability triggered by reconfinement?

- Trigger reconfinement by varying steepness
- Post-shock flow lines converge against the axis
- Connectivity is re-established.
Conclusions

- Causal connectivity of supersonic jet is lost in steeply declining atmospheres, critical case: $p \propto z^{-2}$
  - Suppresses global instabilities, thus no jet disruption
- Novel approach to obtain approximate stationary RMHD equilibria of non-cylindrical jets and follow non-linear evolution of 3D Kink instabilities
- Numerical simulations demonstrate increasing stability for steeper pressure profiles
- Axial core can still become unstable and dissipate magnetic energy – Origin of continuous emission?
- Jet reconfinement due to varying atmosphere can re-establish connectivity “within galaxy” for low power FR1 jets
  - Prospect: Can reconfinement thus trigger global instability?
Thank you!