Rarefaction waves in magnetized astrophysical jets

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Pressure variations in the environment of astrophysical jets induce rarefaction waves propagating inside the jet. Their implications on the jet dynamics will be discussed, focusing on the bulk acceleration and its efficiency in GRB and AGN jets.

Subject :	:	oral
Topics	:	Astrophysics

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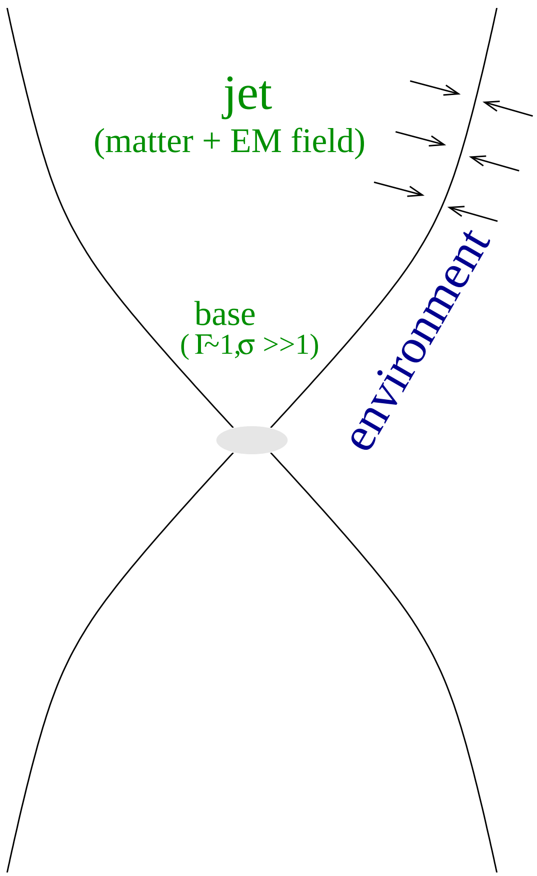
Outline

- “standard” magnetic acceleration (related to collimation)
- rarefaction acceleration
- models – application to GRBs – discussion for AGNs

Magnetized outflows



- Extracted energy per time $\dot{\mathcal{E}}$ mainly in the form of Poynting flux (magnetic fields tap the rotational energy of the compact object or disk)

$$\dot{\mathcal{E}} = \frac{c}{4\pi} \underbrace{\frac{r}{r_{\rm lc}} B_p}_{E} \; B_\phi \times (\text{ area }) \approx \frac{c}{2} B^2 r^2$$

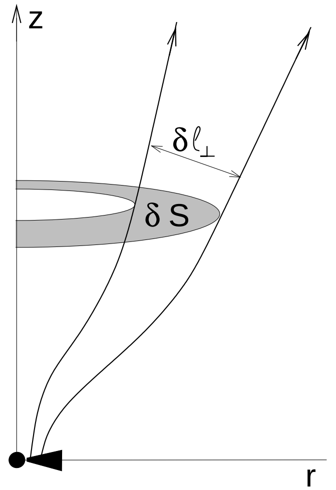
- Ejected mass per time $\dot{M}$

- The $\mu \equiv \dot{\mathcal{E}}/\dot{M}c^2$ gives the maximum possible bulk Lorentz factor of the flow

- Magnetohydrodynamics: matter (velocity, density, pressure) + large scale electromagnetic field

"Standard" model for magnetic acceleration

☞ component of the momentum equation



$$\gamma\rho_0(\boldsymbol{V}\cdot\nabla)\,(\gamma w\boldsymbol{V}) = -\nabla p + J^0\boldsymbol{E} + \boldsymbol{J}\times\boldsymbol{B}$$

along the flow (wind equation): $\gamma \approx \mu - \mathcal{F}$

where $\mathcal{F} \propto r^2 n\gamma V_p = r^2\times$ mass flux

since mass flux $\times\delta S =$ const,

$\mathcal{F} \propto r^2/\delta S \propto r/\delta\ell_\perp$

acceleration requires the separation between streamlines to increase faster than the cylindrical radius

the collimation-acceleration paradigm:
$\mathcal{F}\downarrow$ through stronger collimation of the inner streamlines relative to the outer ones (differential collimation)

☞ transfield component of the momentum equation

$$\frac{\gamma^2 r}{\mathcal{R}} \approx \frac{\left(\dfrac{2I}{\Omega B_p r^2}\right)^2 r \nabla_\perp \ln \left|\dfrac{I}{\gamma}\right|}{1 + \dfrac{4\pi w \rho_0 u_p^2}{B_p^2}\dfrac{r_{\rm lc}^2}{r^2}} - \gamma^2 \frac{r_{\rm lc}^2}{r^2} \nabla_\perp r,$$ with $\nabla_\perp \sim \frac{1}{r}$, $r_{\rm lc} = \frac{c}{\Omega}$,

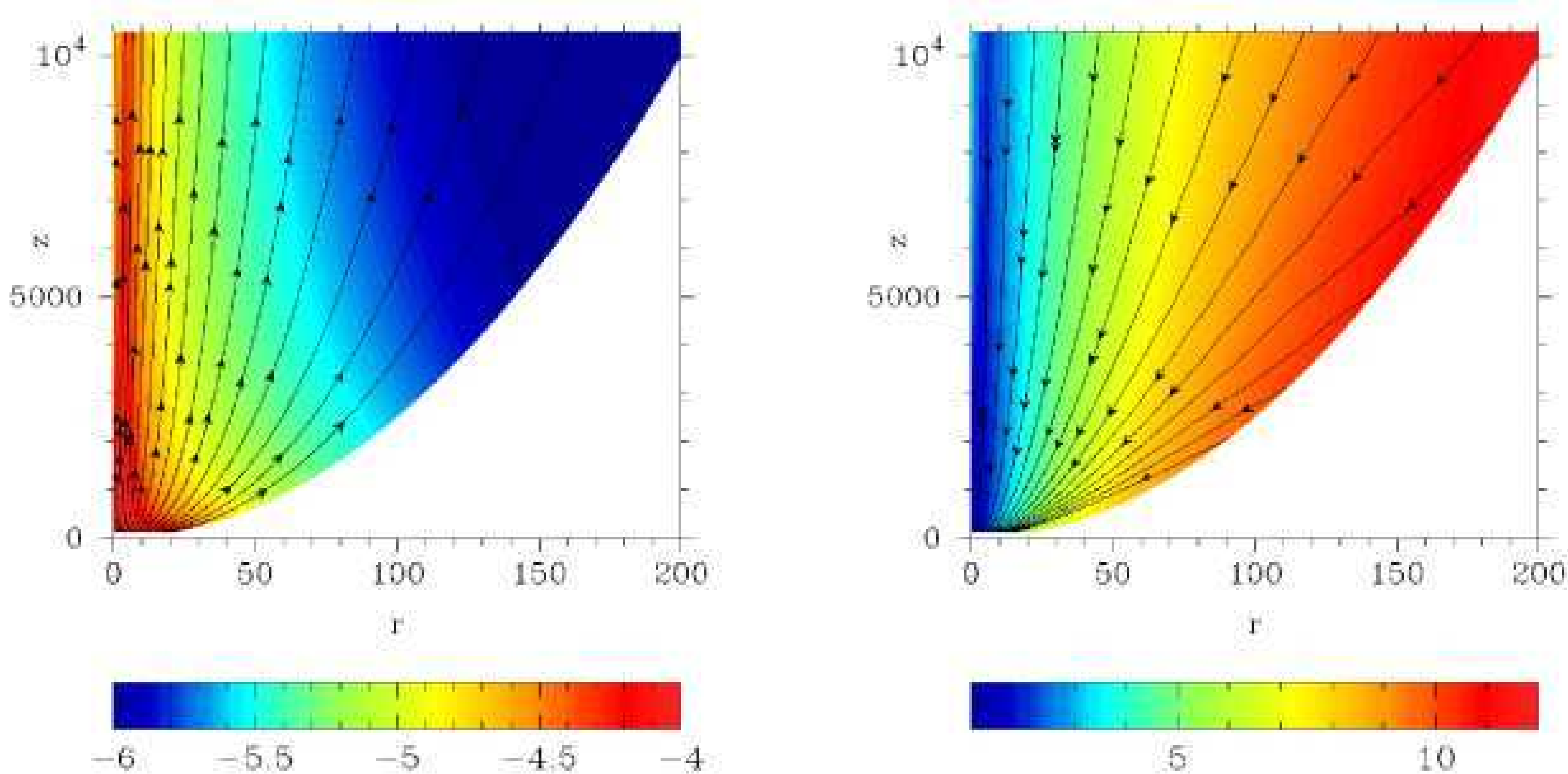
simplifies to $$\underbrace{\frac{\gamma^2 r}{\mathcal{R}}}_{inertia} \approx \underbrace{1}_{EM} - \underbrace{\gamma^2 \frac{r_{\rm lc}^2}{r^2}}_{centrifugal}$$

• if centrifugal negligible then $\gamma \approx z/r$ (since $\mathcal{R}^{-1} \approx -\frac{d^2 r}{dz^2} \approx \frac{r}{z^2}$)
power-law acceleration regime
(for parabolic shapes $z \propto r^a$, γ is a power of r)

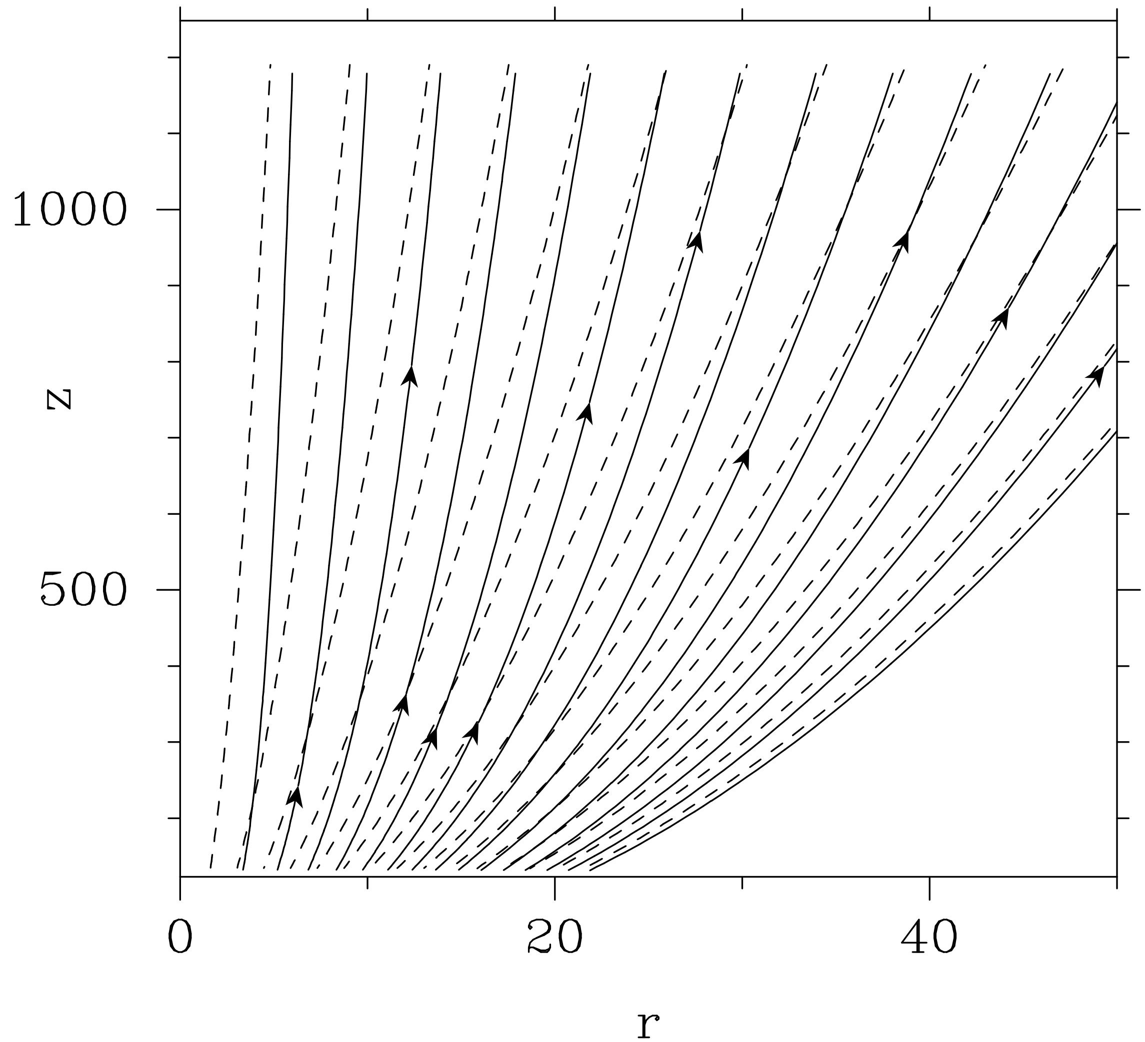
• if inetria negligible then $\gamma \approx r/r_{\rm lc}$ linear acceleration regime

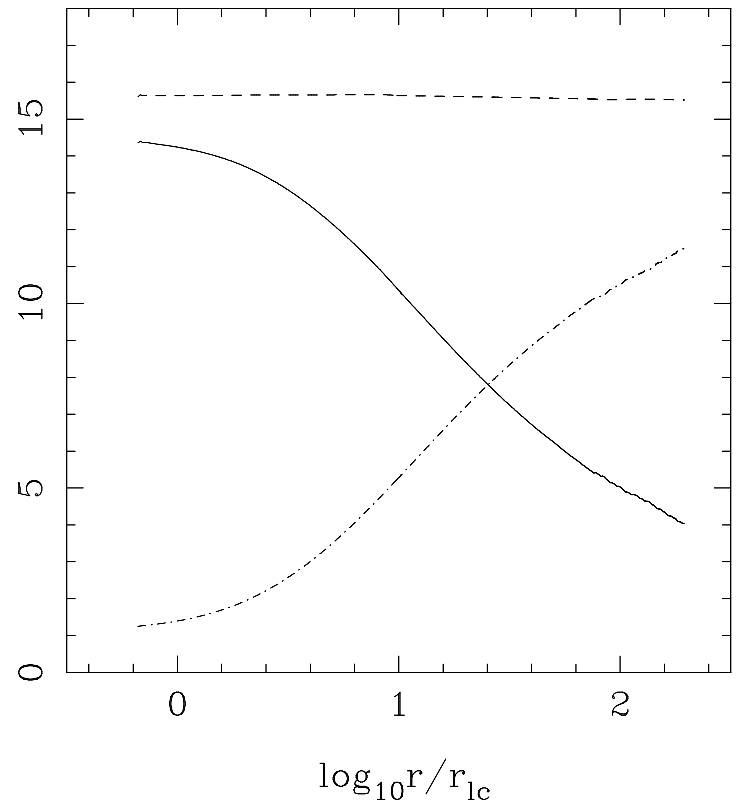
• if electromagnetic negligible then ballistic regime

Simulations of relativistic jets
Komissarov, Barkov, Vlahakis, & Königl (2007)



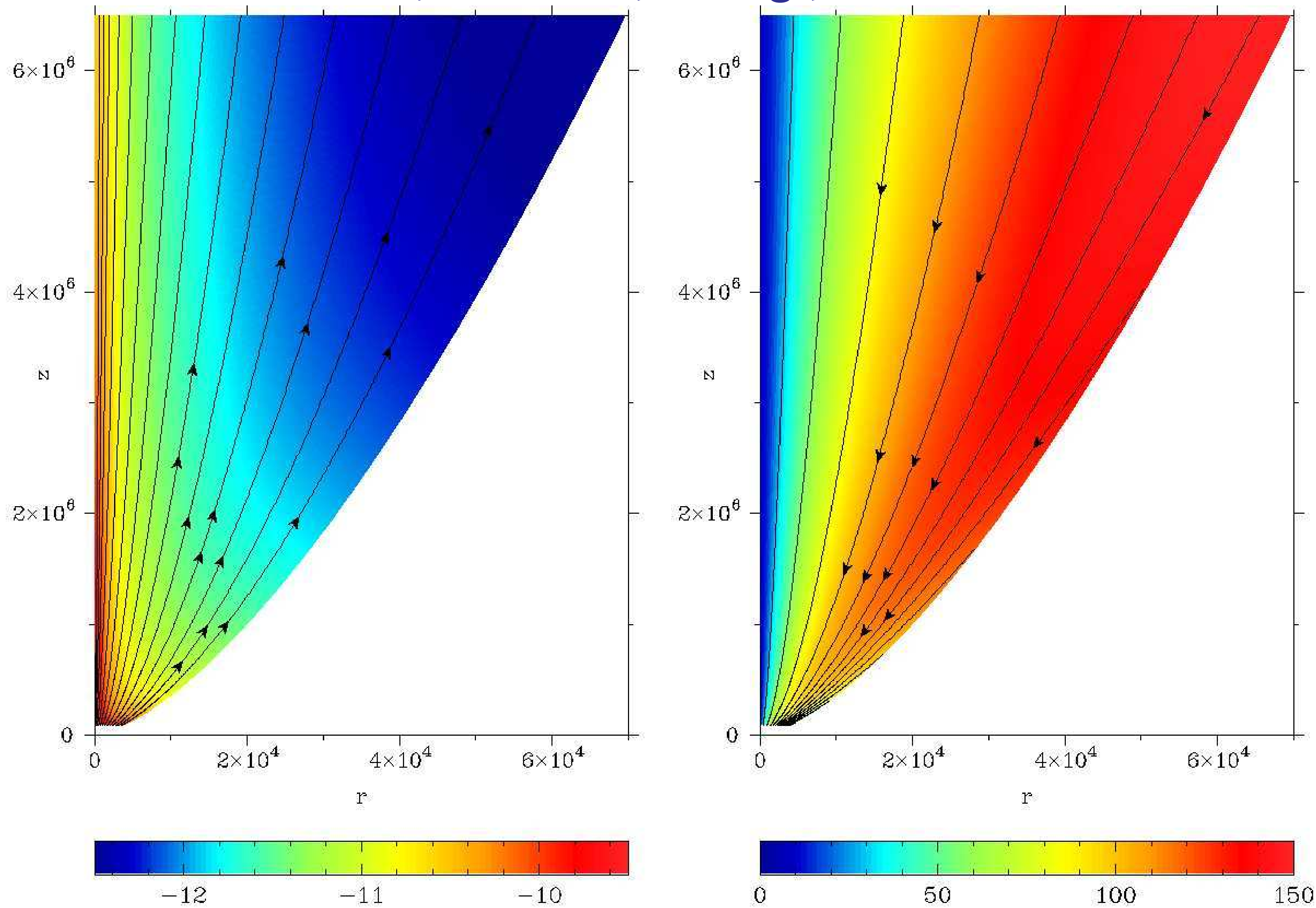
Left panel shows density (colour) and magnetic field lines. Right panel shows the Lorentz factor (colour) and the current lines.

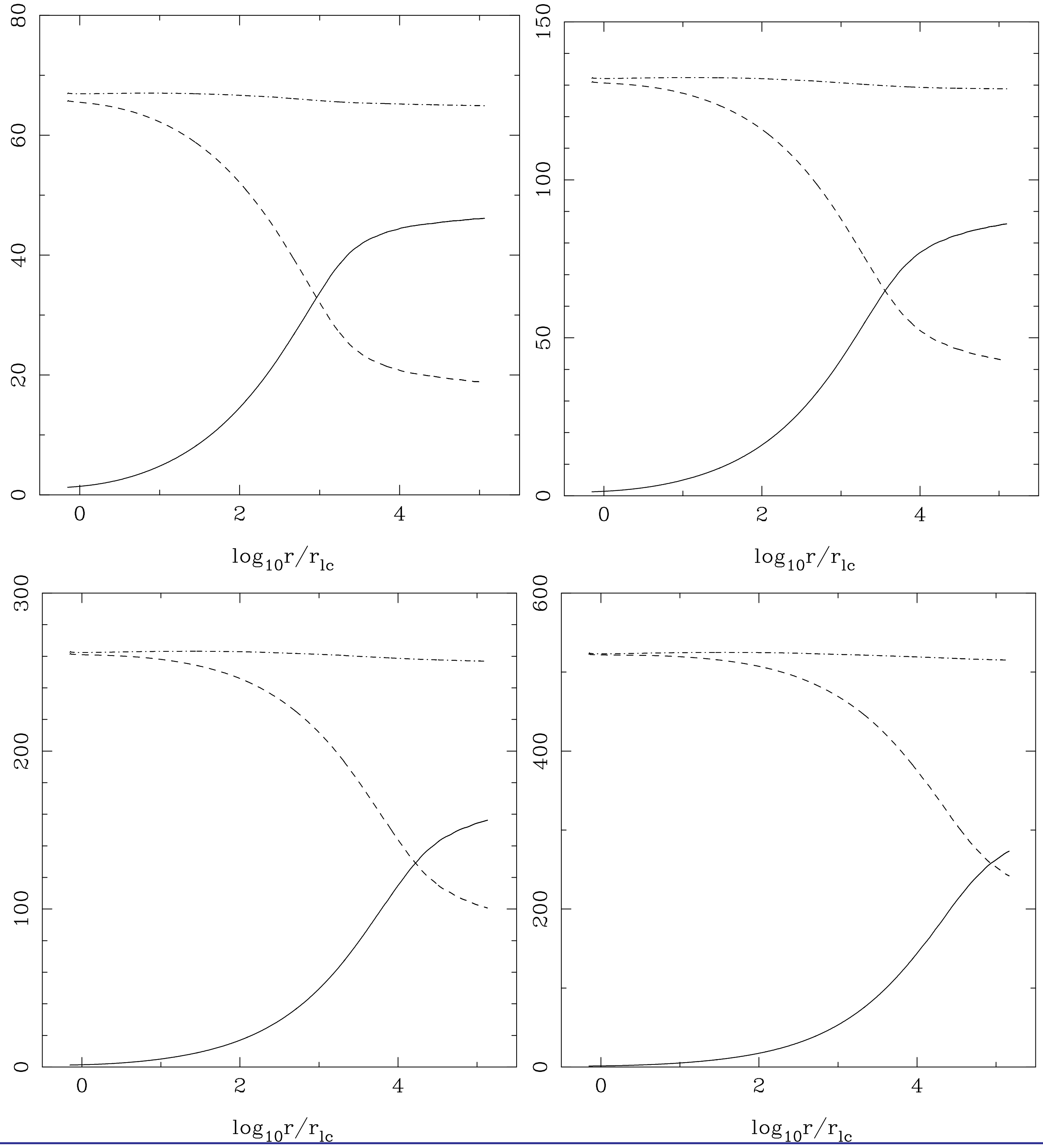

1000
500
z
0
20
40
r



$\gamma\sigma$ (solid line), μ (dashed line) and γ (dash-dotted line) along a magnetic field line as a function of cylindrical radius

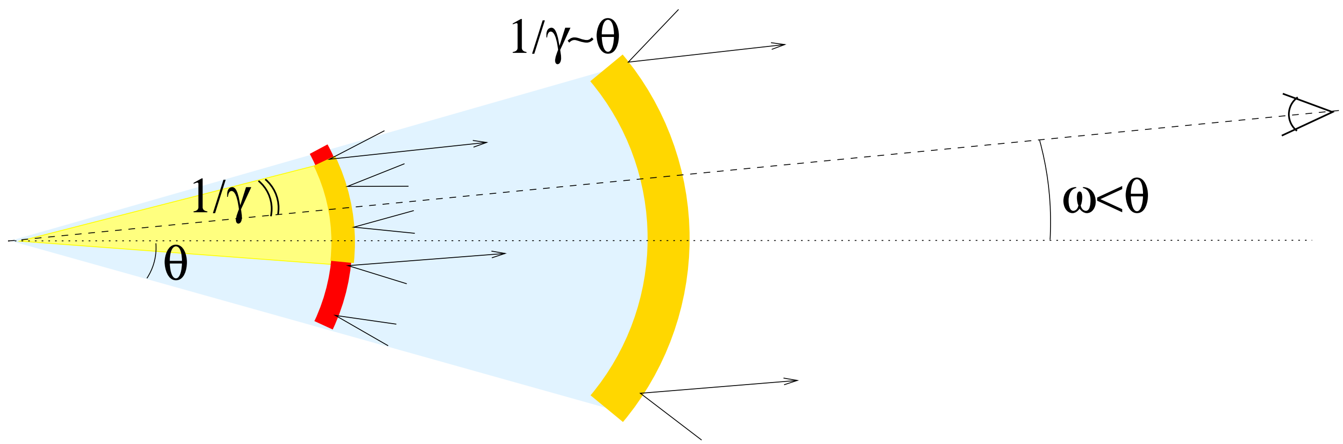
Komissarov, Vlahakis, Königl, & Barkov 2009





80
60
40
20
0
0
2
4
$\log_{10} r/r_{lc}$
150
100
50
0
0
2
4
$\log_{10} r/r_{lc}$
300
200
100
0
0
2
4
$\log_{10} r/r_{lc}$
600
400
200
0
0
2
4
$\log_{10} r/r_{lc}$

Caveat: $\gamma\vartheta \sim 1$ (for high γ)



During the afterglow γ decreases

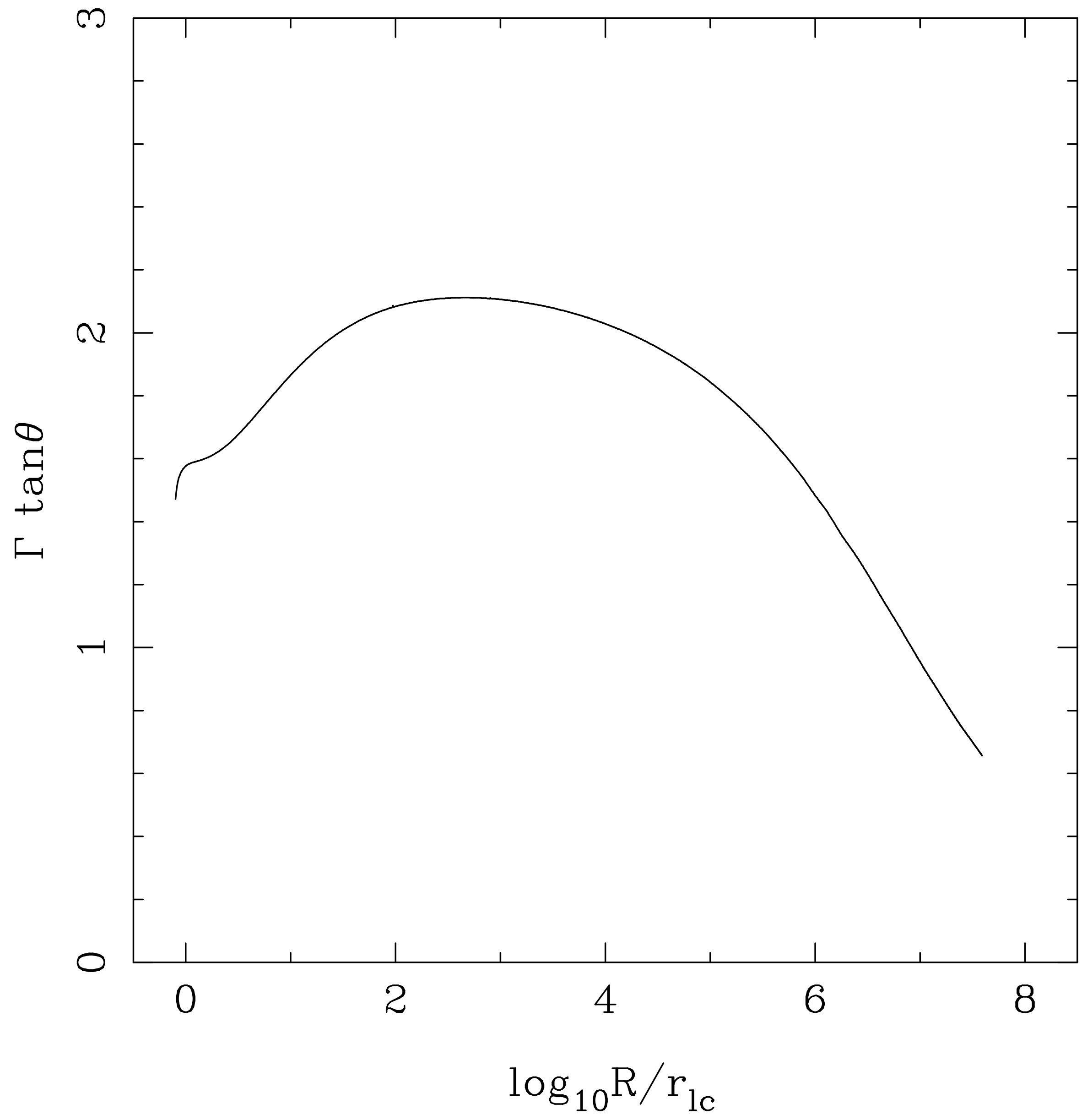
When $1/\gamma > \vartheta$ the observed flux decreases faster with time

- with $\gamma\vartheta \sim 1$ very narrow jets ($\vartheta < 1^\circ$ for $\gamma > 100$) $\longrightarrow$ early breaks or no breaks at all

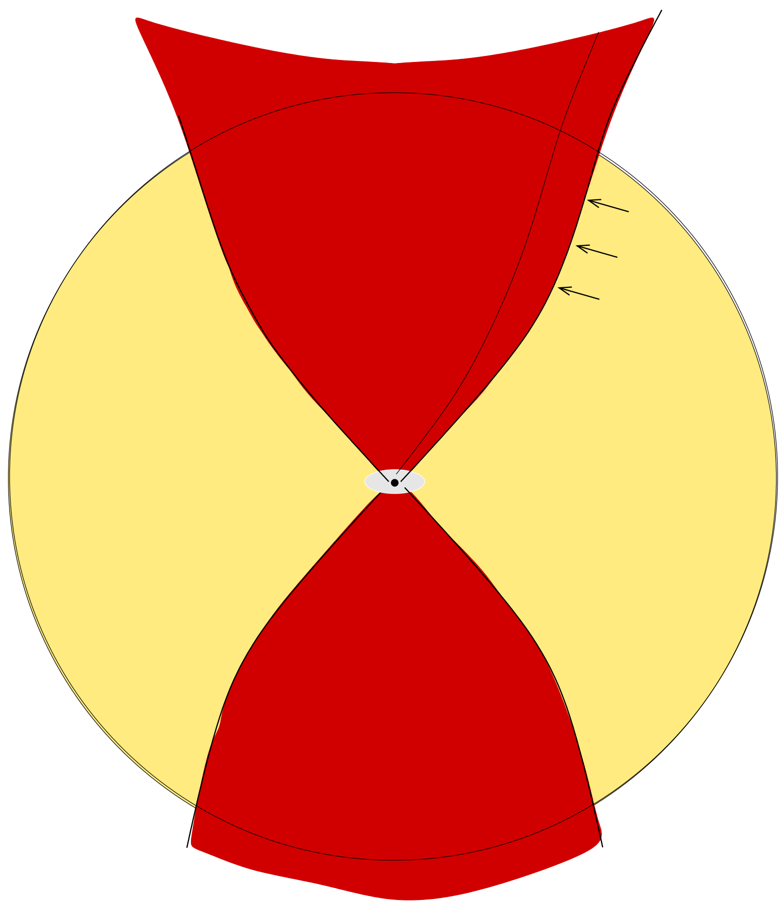
- this is a result of causality (across jet): outer lines need to know that there is space to expand

- equivalent to $\mathcal{R} \approx \gamma^2 r$ (transfield force balance)

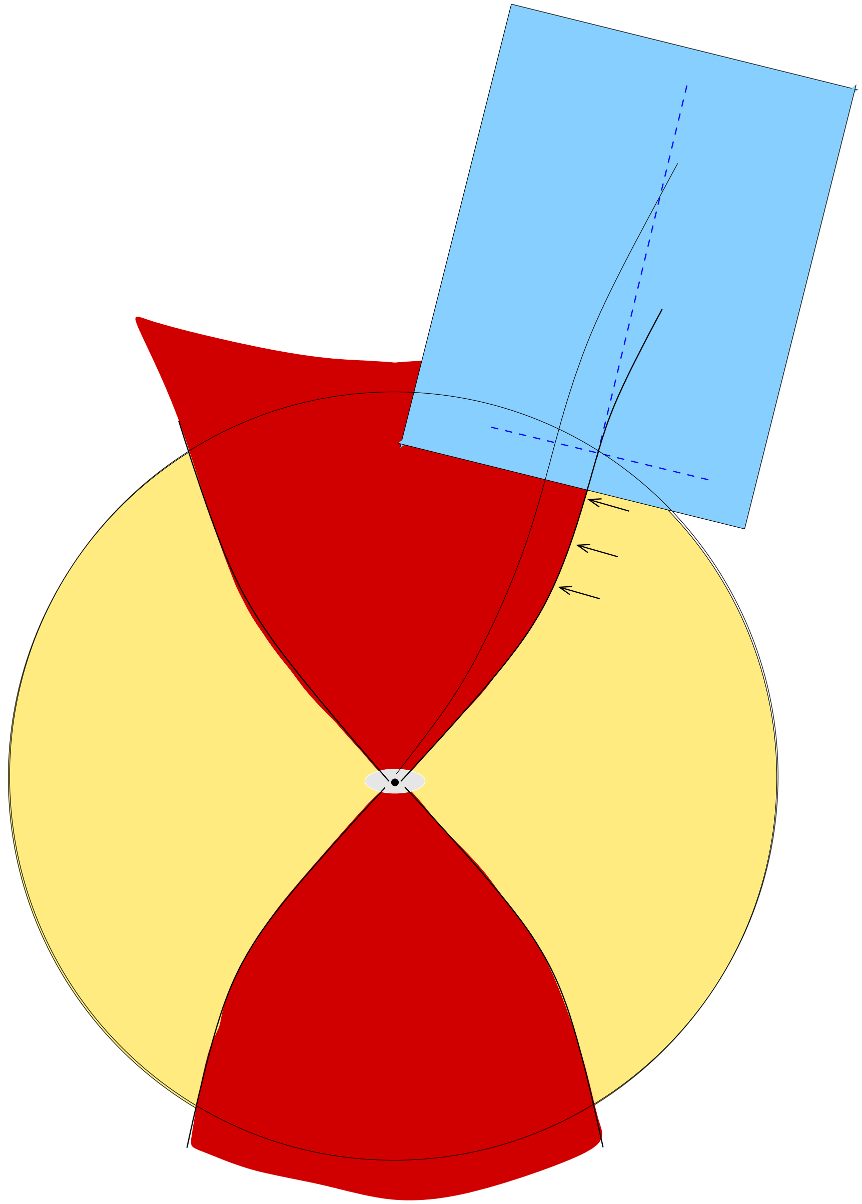
- Mach cone half-opening θ_m should be $> \vartheta$
  With $\sin\theta_m = \dfrac{\gamma_f c_f}{\gamma V_p} \approx \dfrac{\sigma^{1/2}}{\gamma}$ the requirement for causality yields $\gamma\vartheta < \sigma^{1/2}$.
  For efficient acceleration ($\sigma \sim 1$ or smaller) we always get $\gamma\vartheta \sim 1$


3
2
1
0
Γ tanθ
0
2
4
6
8
log10R/rlc

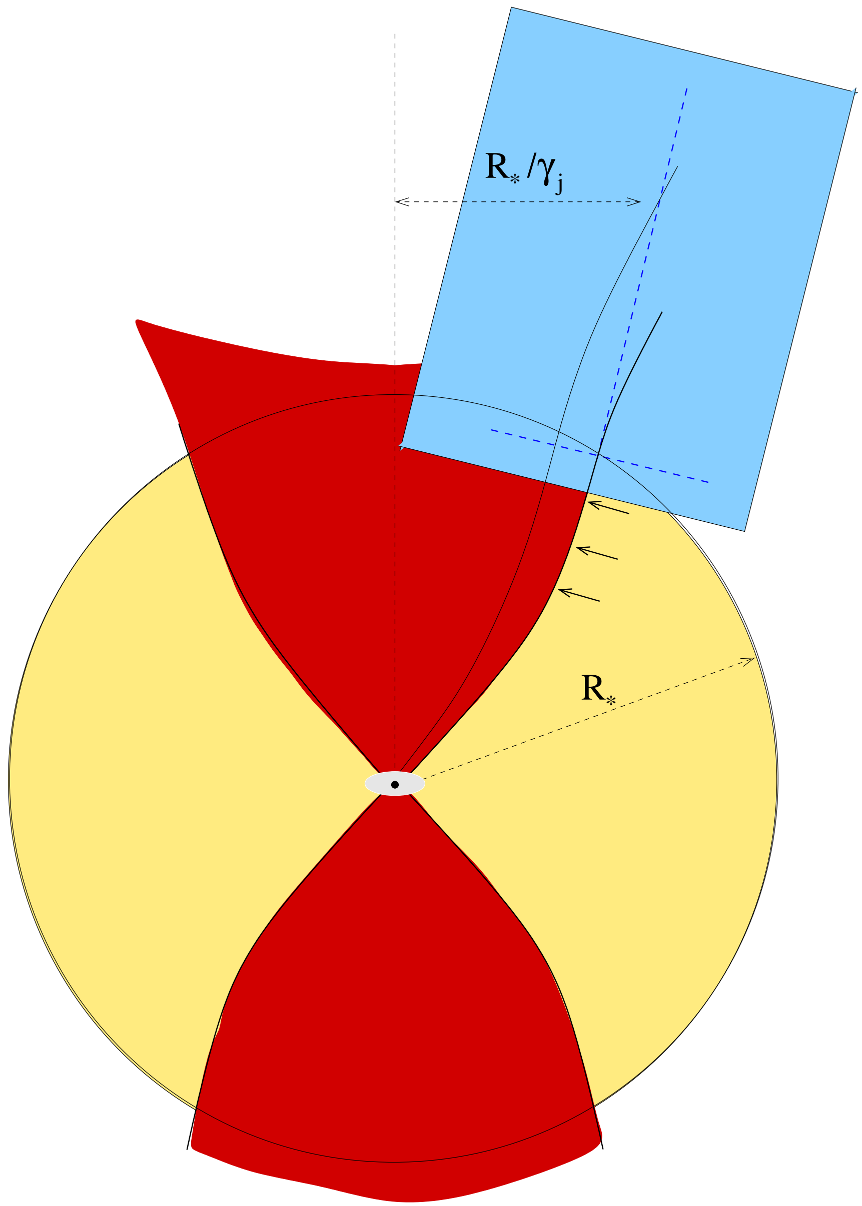
Rarefaction acceleration



Rarefaction acceleration

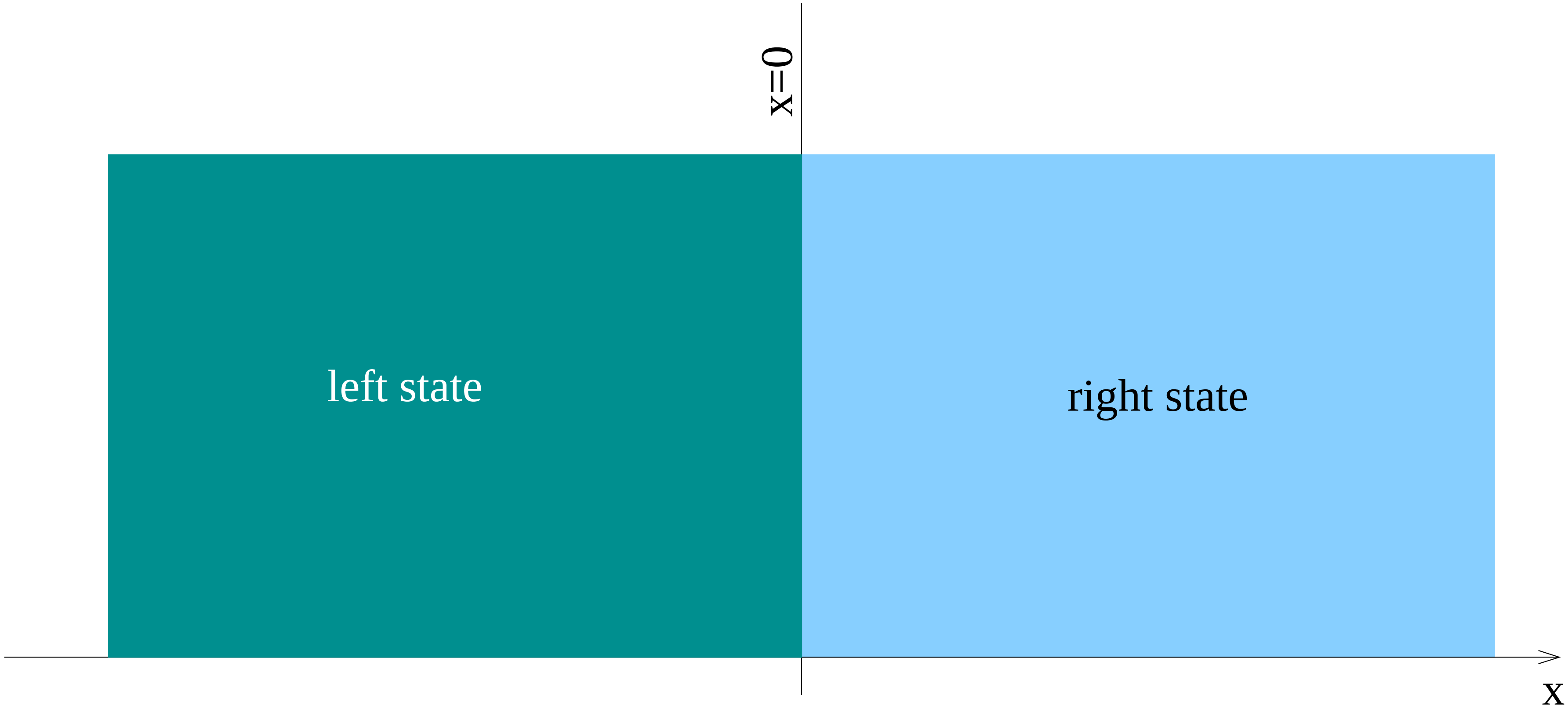


Rarefaction acceleration



Rarefaction simple waves

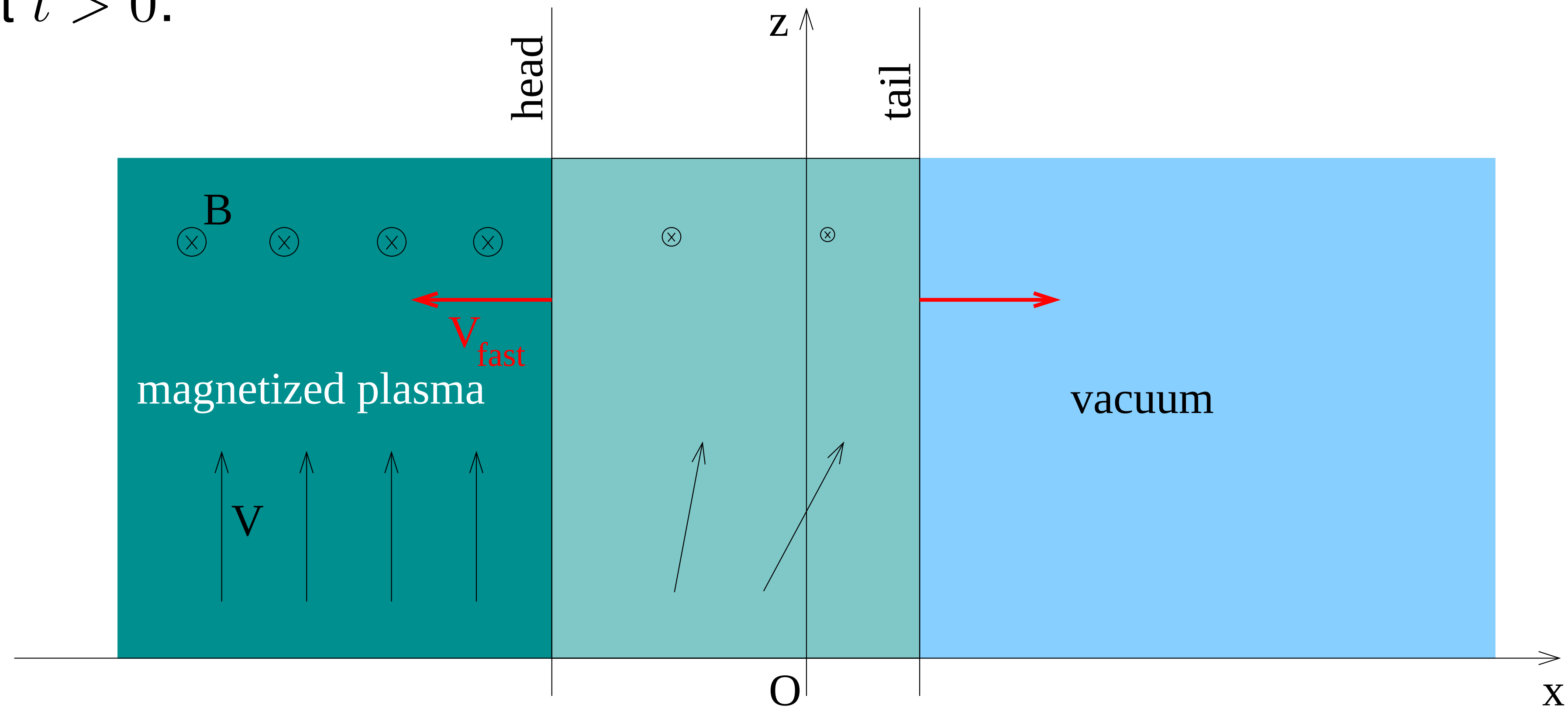
At $t = 0$ two uniform states are in contact:



This Riemann problem allows self-similar solutions that depend only on $\xi = x/t$.

- when $\rho_R/\rho_L = 0$ simple rarefaction wave

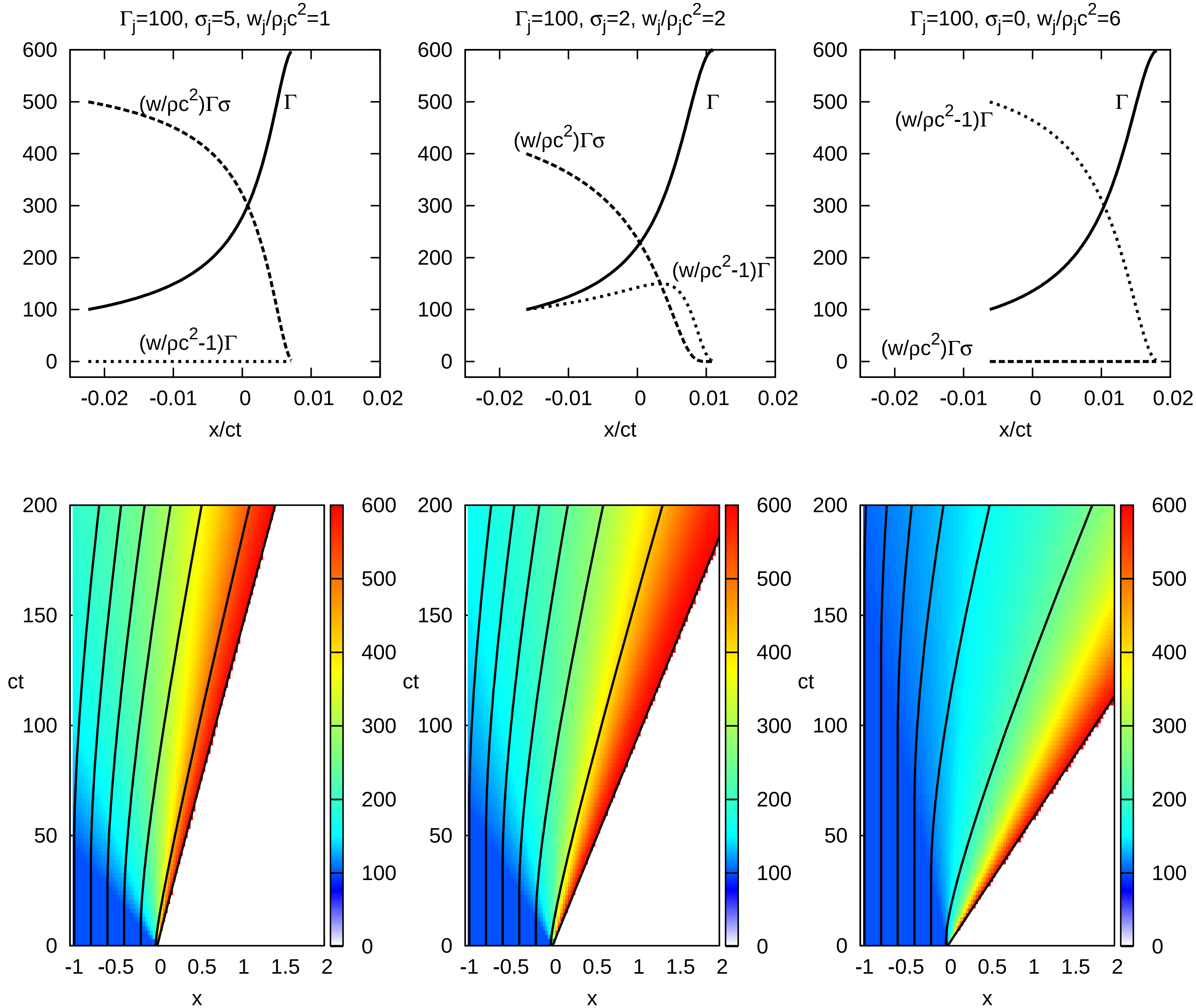
At $t > 0$:



for the cold case the Riemann invariants imply

$$v_x = \frac{1}{\gamma_j}\frac{2\sigma_j^{1/2}}{1+\sigma_j}\left[1-\left(\frac{\rho}{\rho_j}\right)^{1/2}\right],\ \gamma = \frac{\gamma_j\,(1+\sigma_j)}{1+\sigma_j\rho/\rho_j}\,,\ \rho = \frac{4\rho_j}{\sigma_j}\sinh^2\left[\frac{1}{3}\mathrm{arcsinh}\left(\sigma_j^{1/2}-\frac{\mu_j}{2}\frac{x}{t}\right)\right]$$

$$V_{head} = -\frac{\sigma_j^{1/2}}{\gamma_j}\,, \qquad V_{tail} = \frac{1}{\gamma_j}\frac{2\sigma_j^{1/2}}{1+\sigma_j}\,, \qquad \Delta\vartheta = V_{tail} < 1/\gamma_i$$

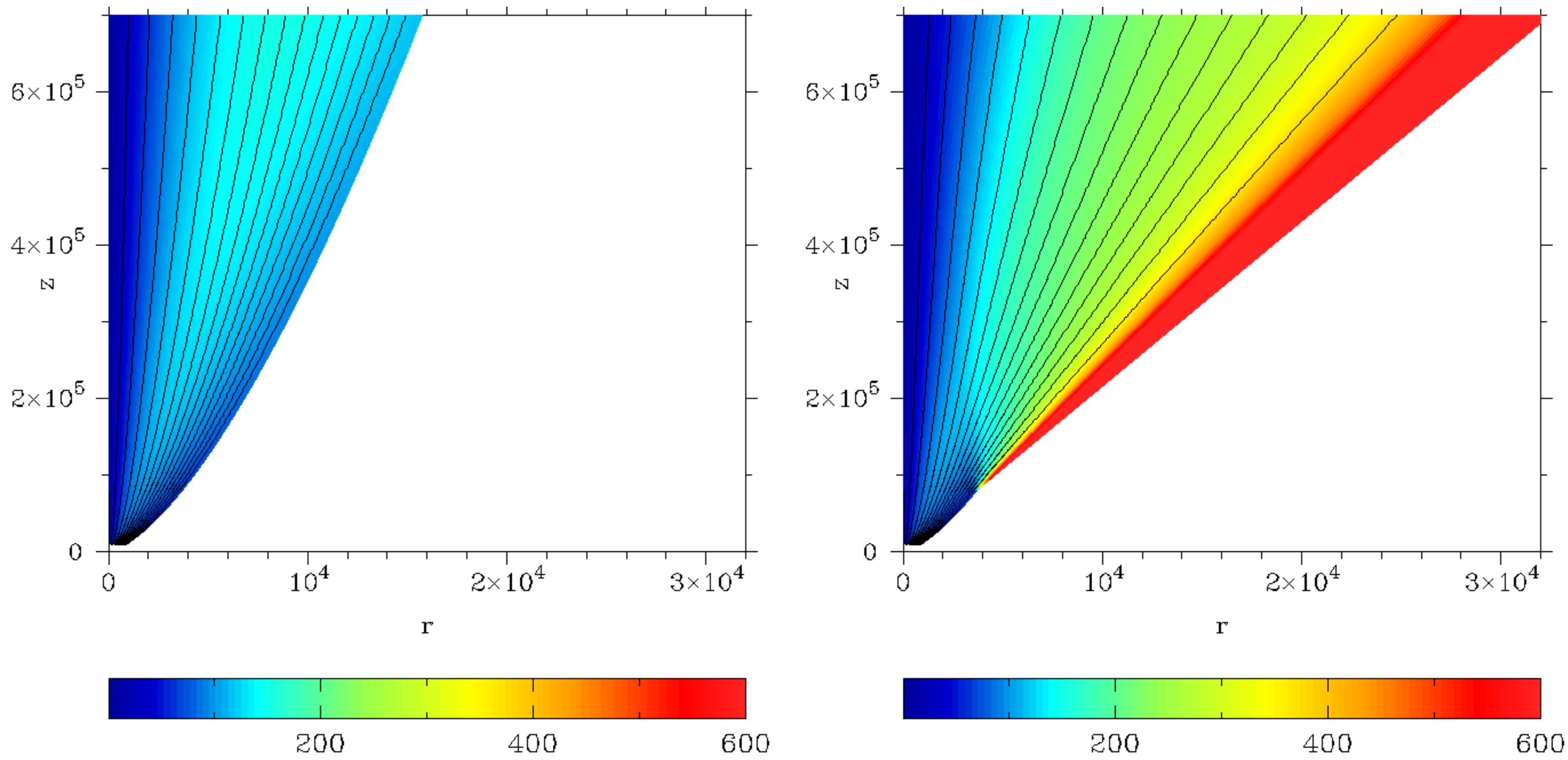


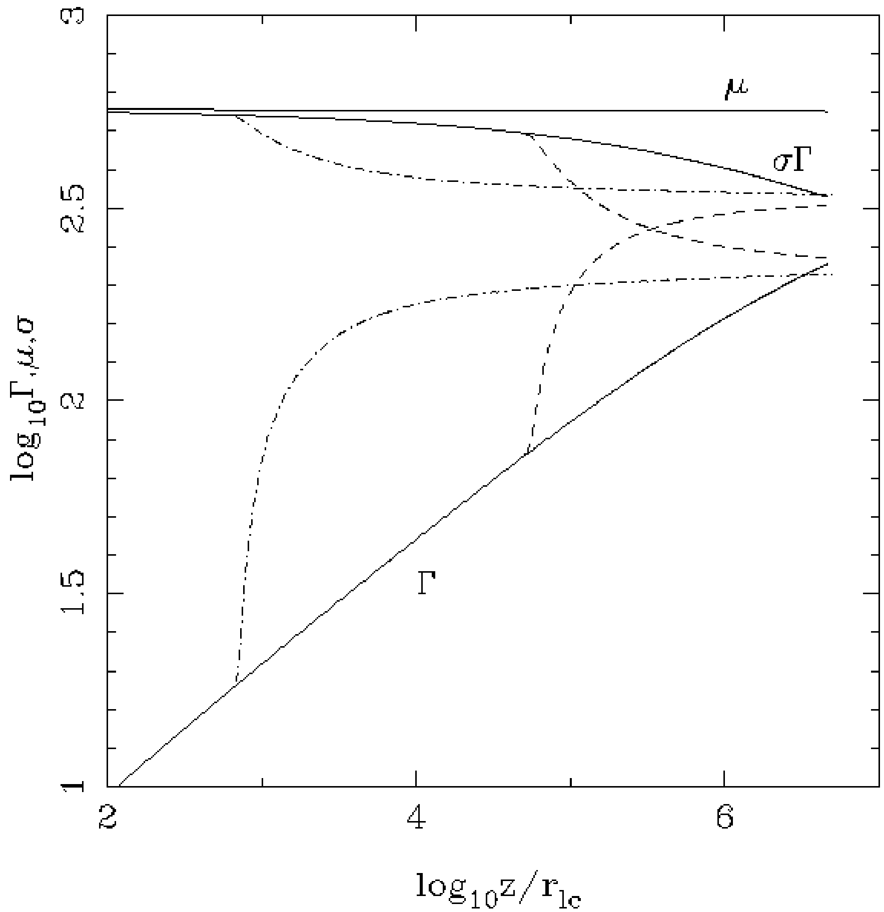
The colour image in the Minkowski diagrams represents the distribution of the Lorentz factor and the contours show the worldlines of various fluid parcels. (see also Aloy & Rezzolla 2006 for HD, Mizuno+2008 for MHD)

Simulation results

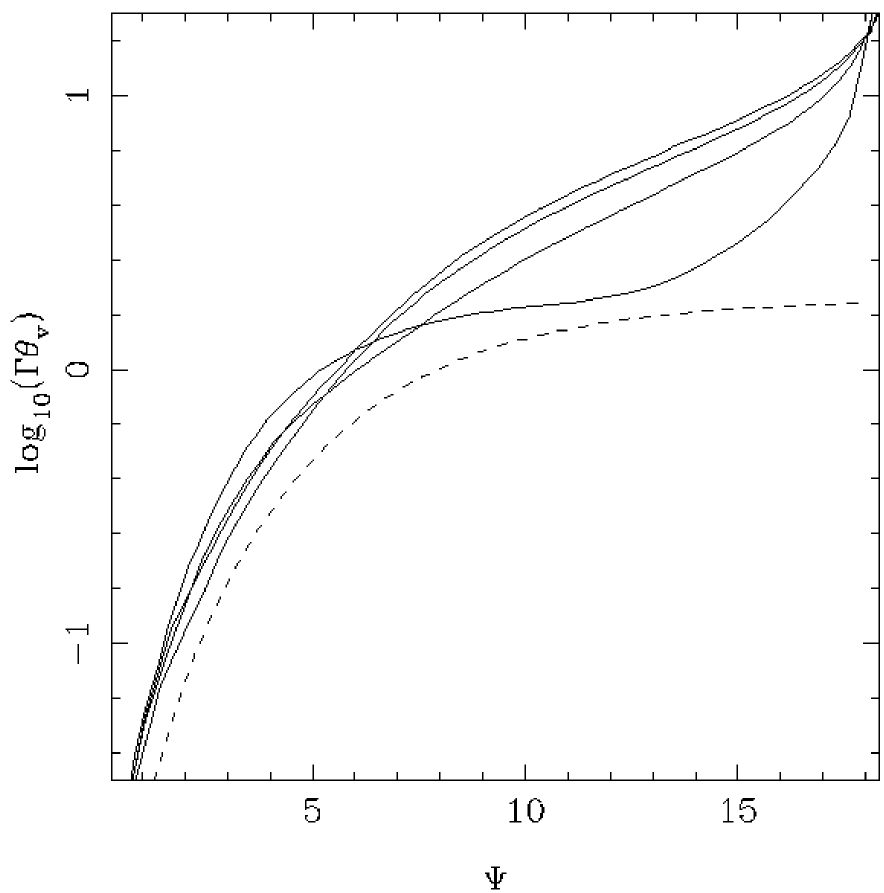
Komissarov, Vlahakis & Königl 2010

(see also Tchekhovskoy, Narayan & McKinney 2010)





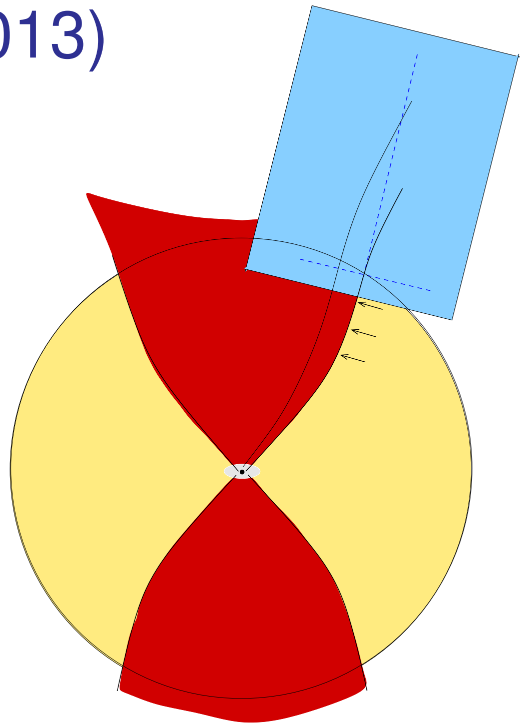
μ
$\sigma\Gamma$
Γ
$\log_{10}\Gamma, \mu, \sigma$
$\log_{10} z/r_{lc}$



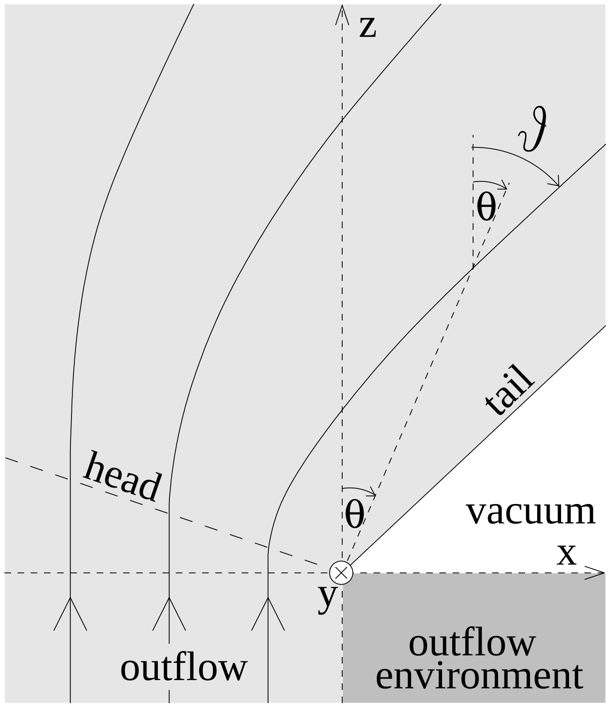
$\log_{10}(\Gamma\theta_v)$
Ψ

Steady-state rarefaction wave

Sapountzis & Vlahakis (2013)



- "flow around a corner"
- planar geometry
- ignoring B_p (nonzero B_y)
- similarity variable x/z (angle θ)
- generalization of the nonrelativistic, hydrodynamic rarefaction (e.g. Landau & Lifshitz)

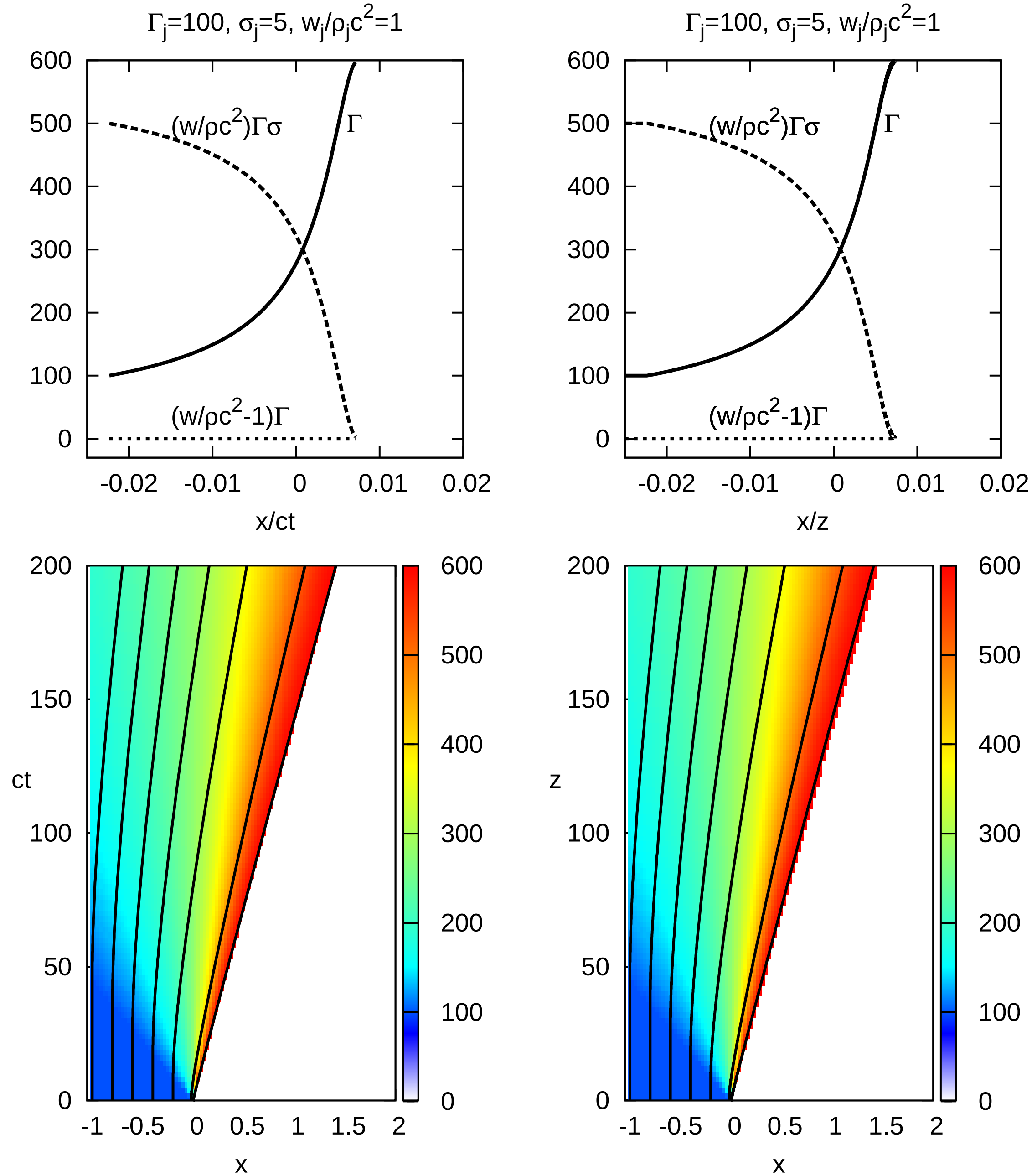


$$\theta_{\rm head} = -\frac{\sigma_j^{1/2}}{\gamma_j}$$

$$\theta_{\rm tail} = \frac{2\sigma_j^{1/2}}{\gamma_j(1+\sigma_j)}$$

$$\sigma = (\sigma_j \gamma_j x_i / z)^{2/3}$$

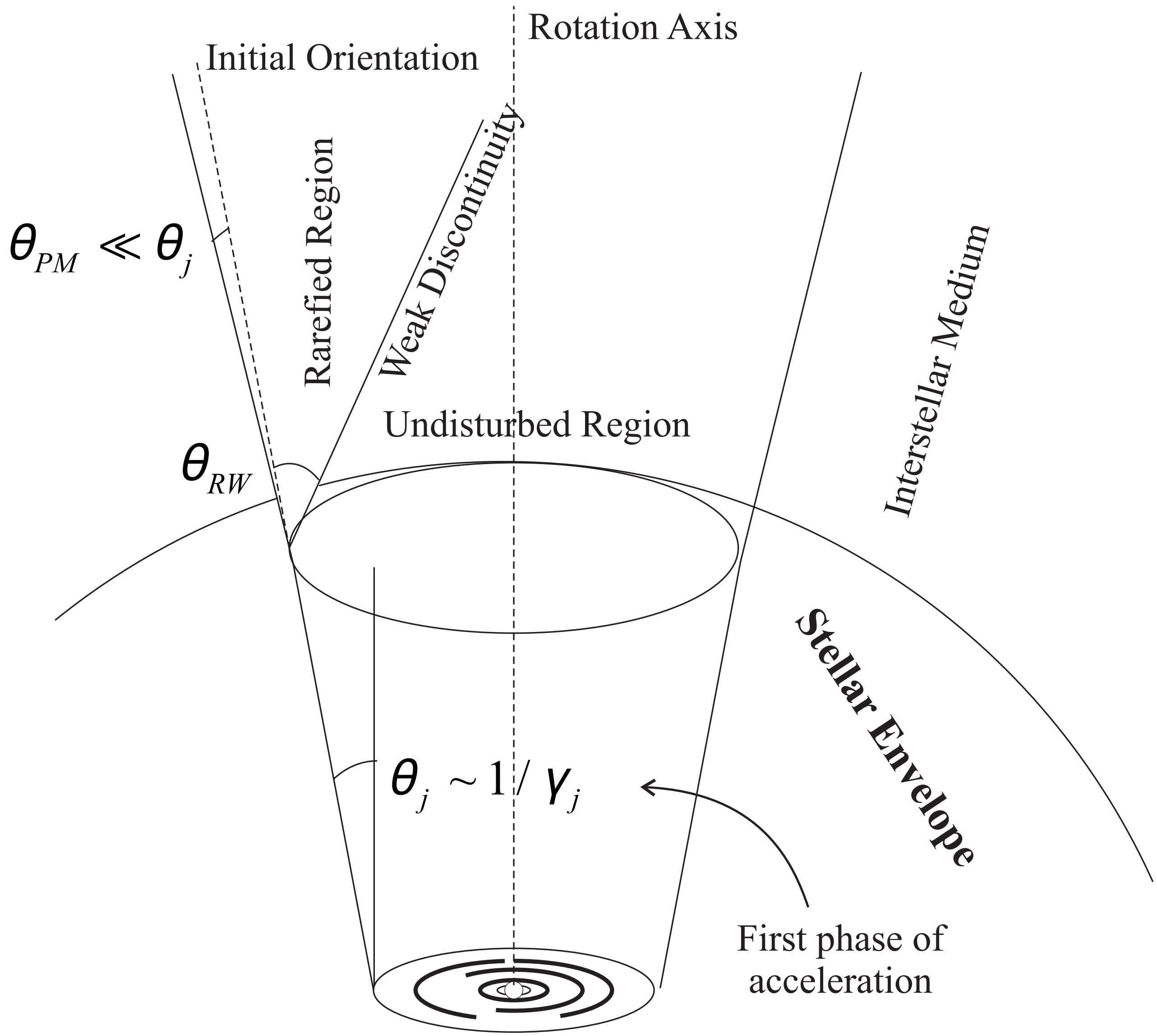
$\sigma = 1$ at $r = \sigma_j \gamma_j |x_i| = 7\times 10^{11} \sigma_j \left(\frac{|x_i|}{R_\star/\gamma_j}\right) \left(\frac{R_\star}{10 R_\odot}\right)$ cm



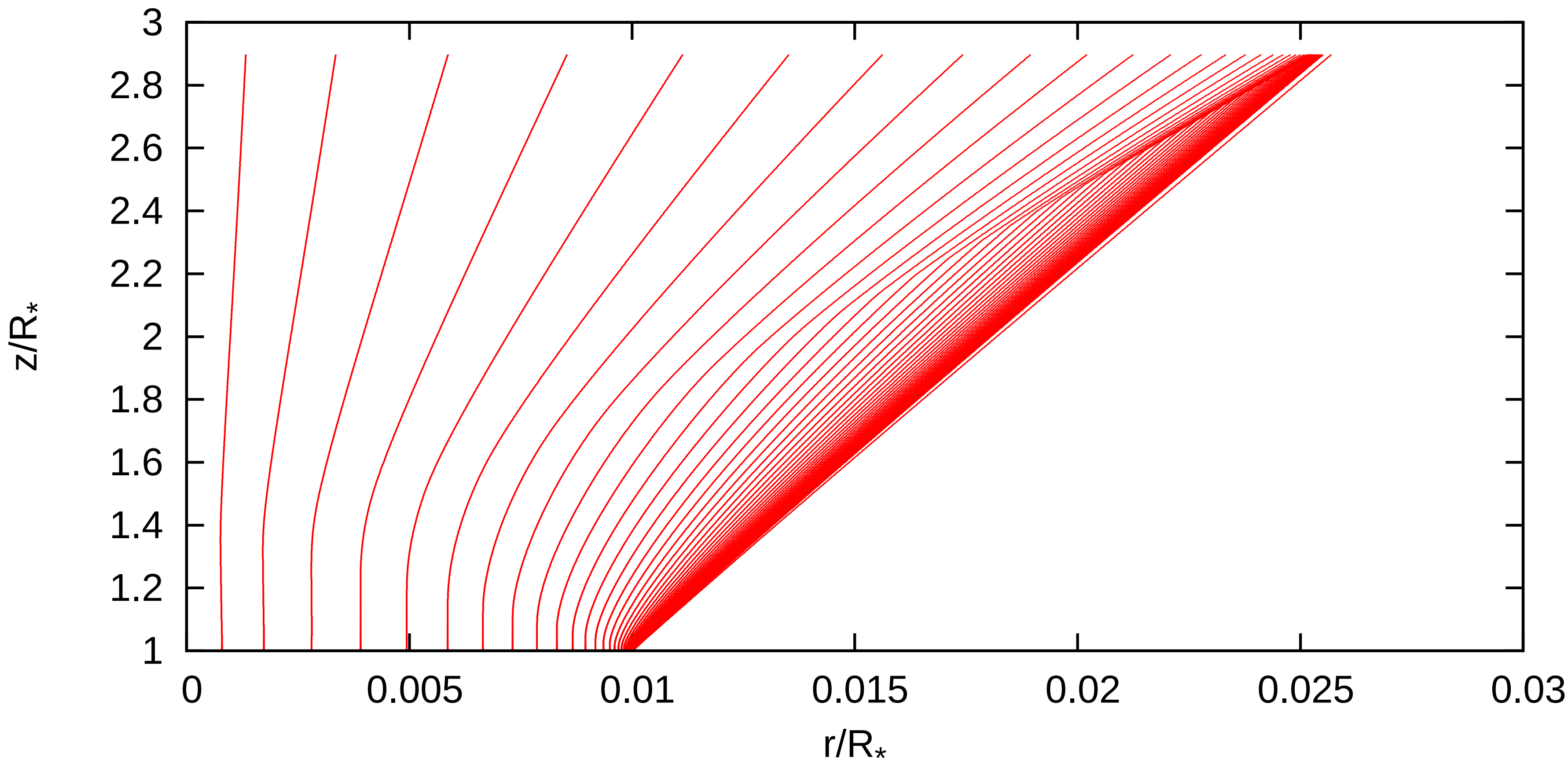
time-dependent (left) and steady-state (right) rarefaction
(similar; $ct \to z$)
(distance unit $= R_\star/\gamma_j \sim 10^{10}$ cm)

Axisymmetric model

Solve steady-state axisymmetric MHD eqs using the method of characteristics (Sapountzis & Vlahakis in preparation)

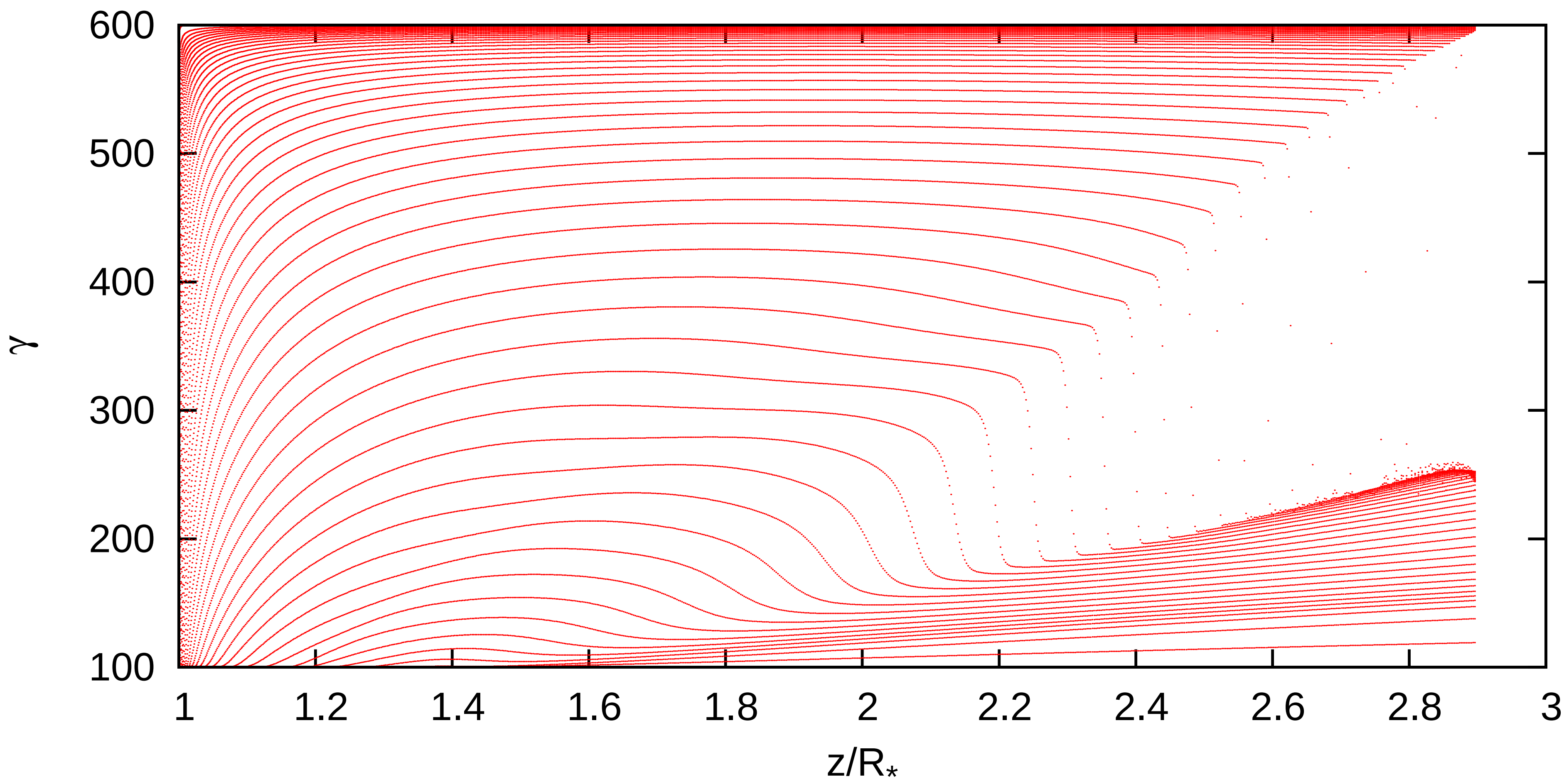


$\gamma_j = 100$, $\sigma_j = 1$, $\rho_{ext} = 0$

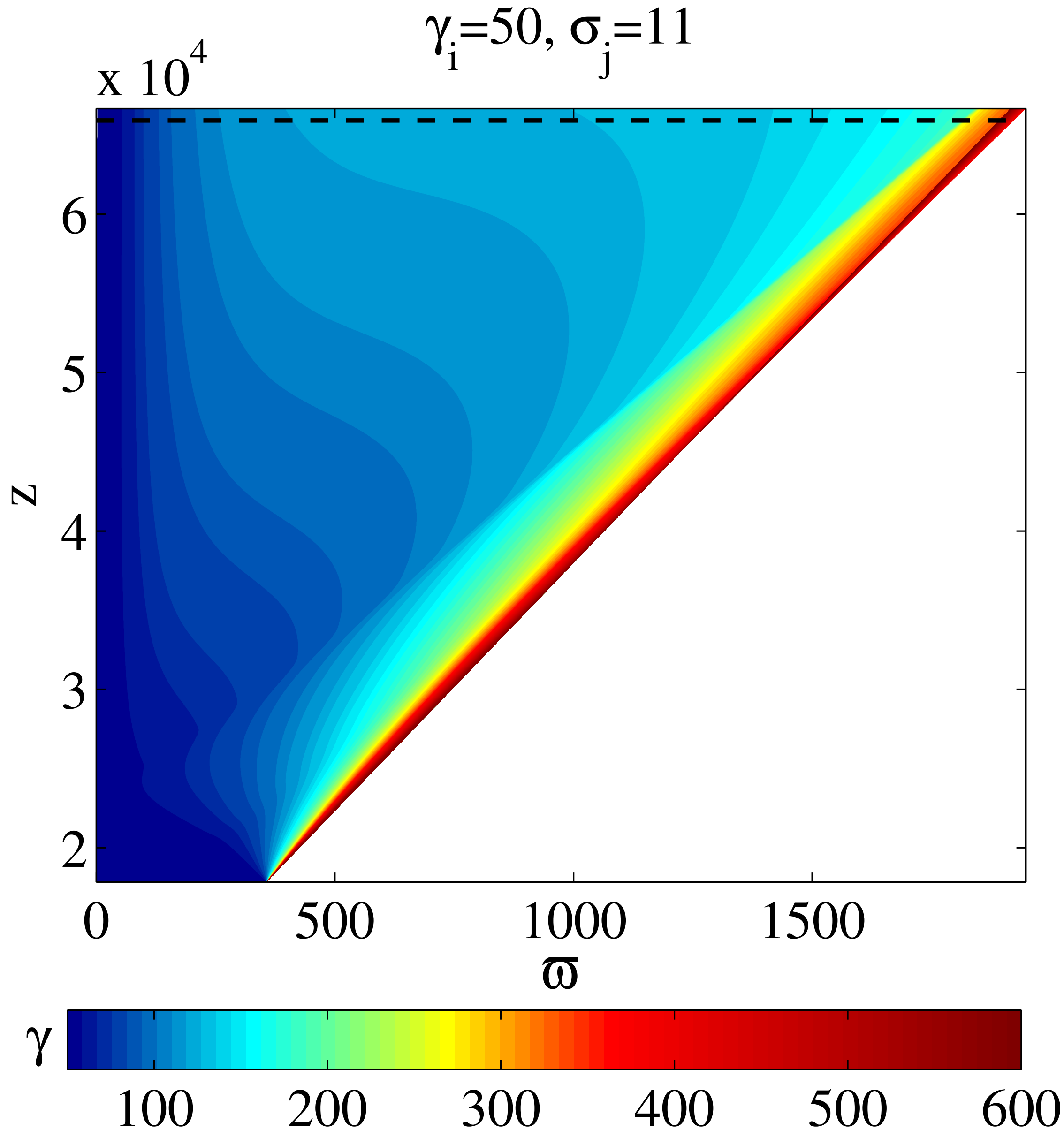


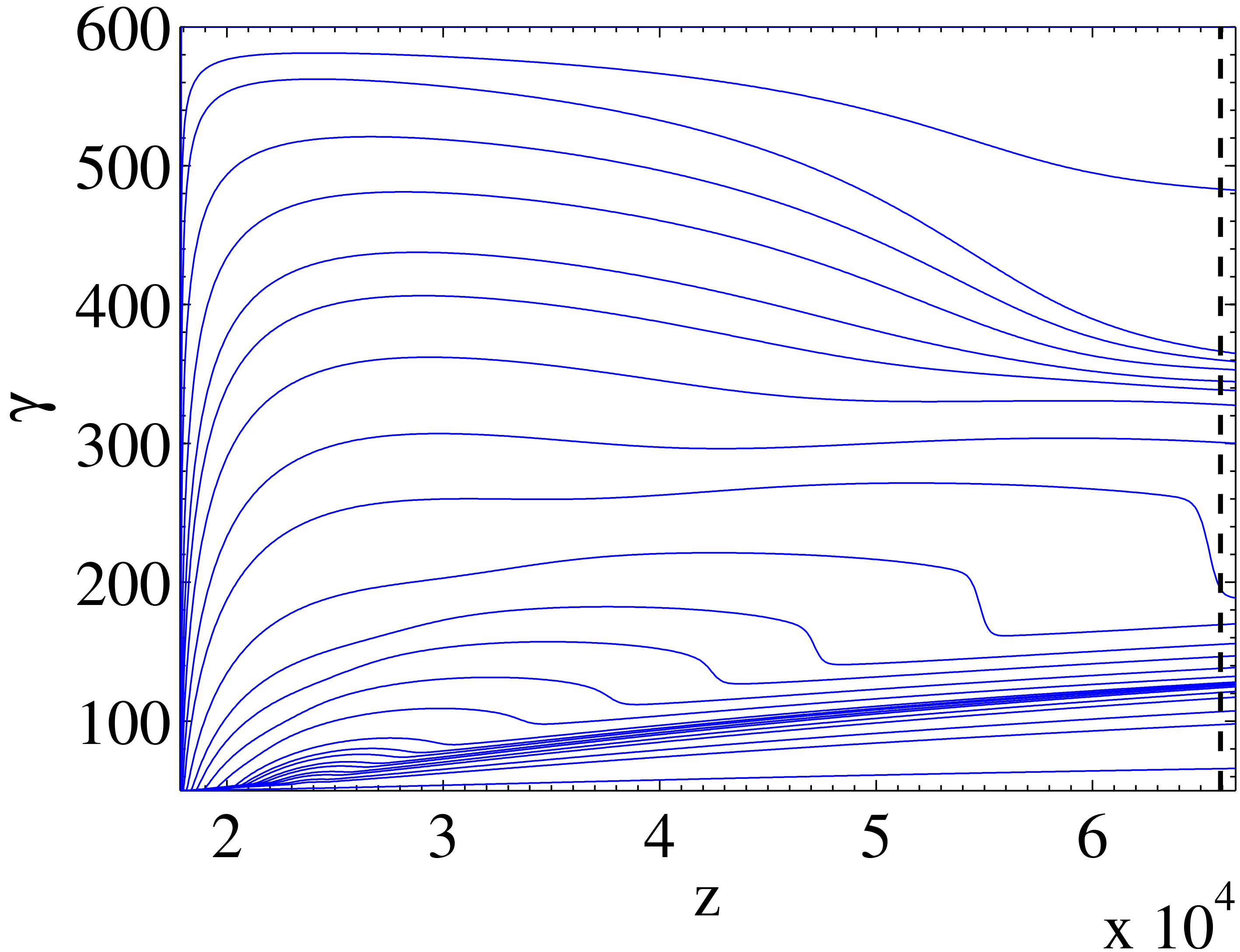
(not in scale!)

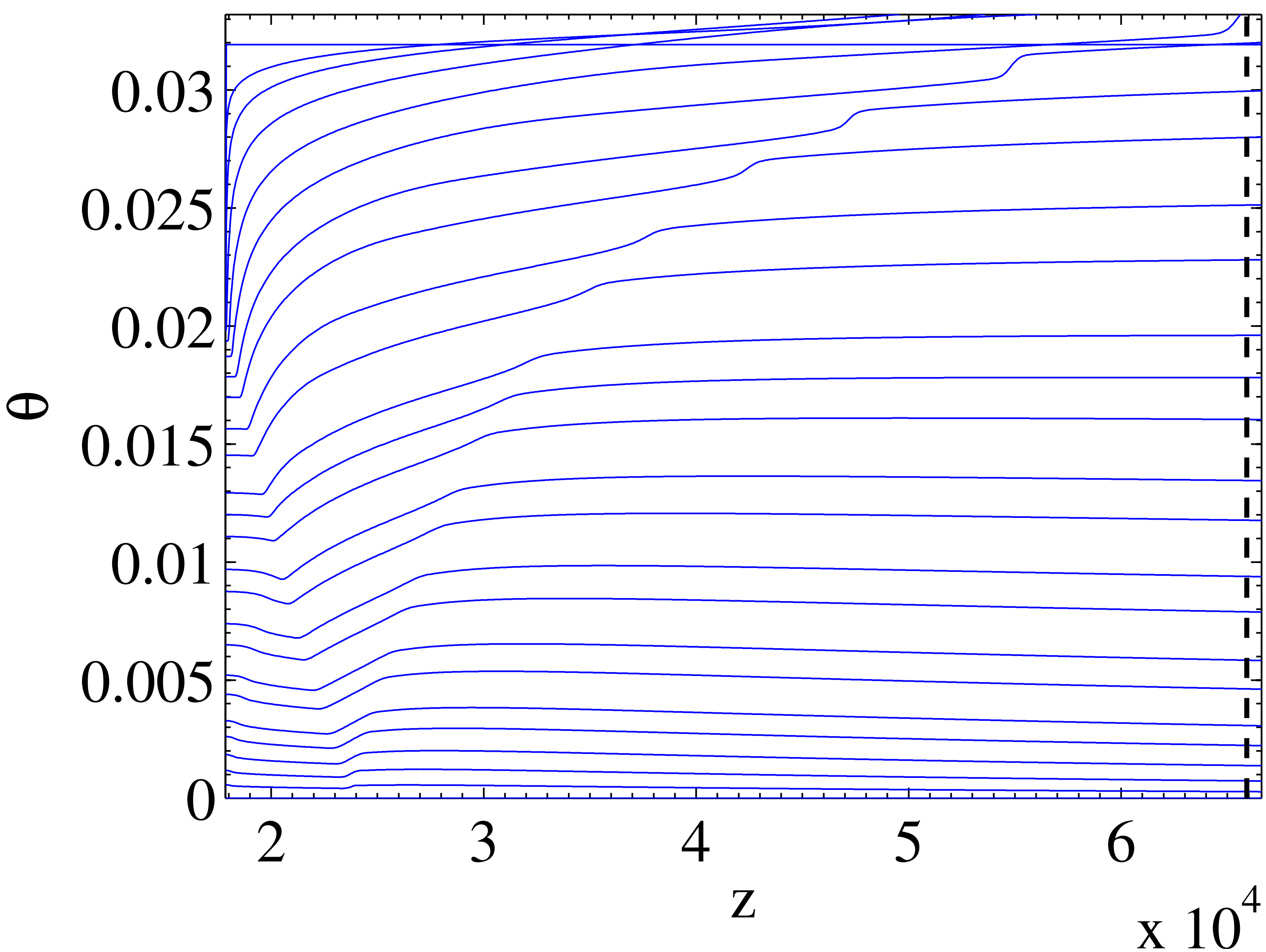
Reflection of the wave from the axis



Reflection causes sudden deceleration – standing shock (?)

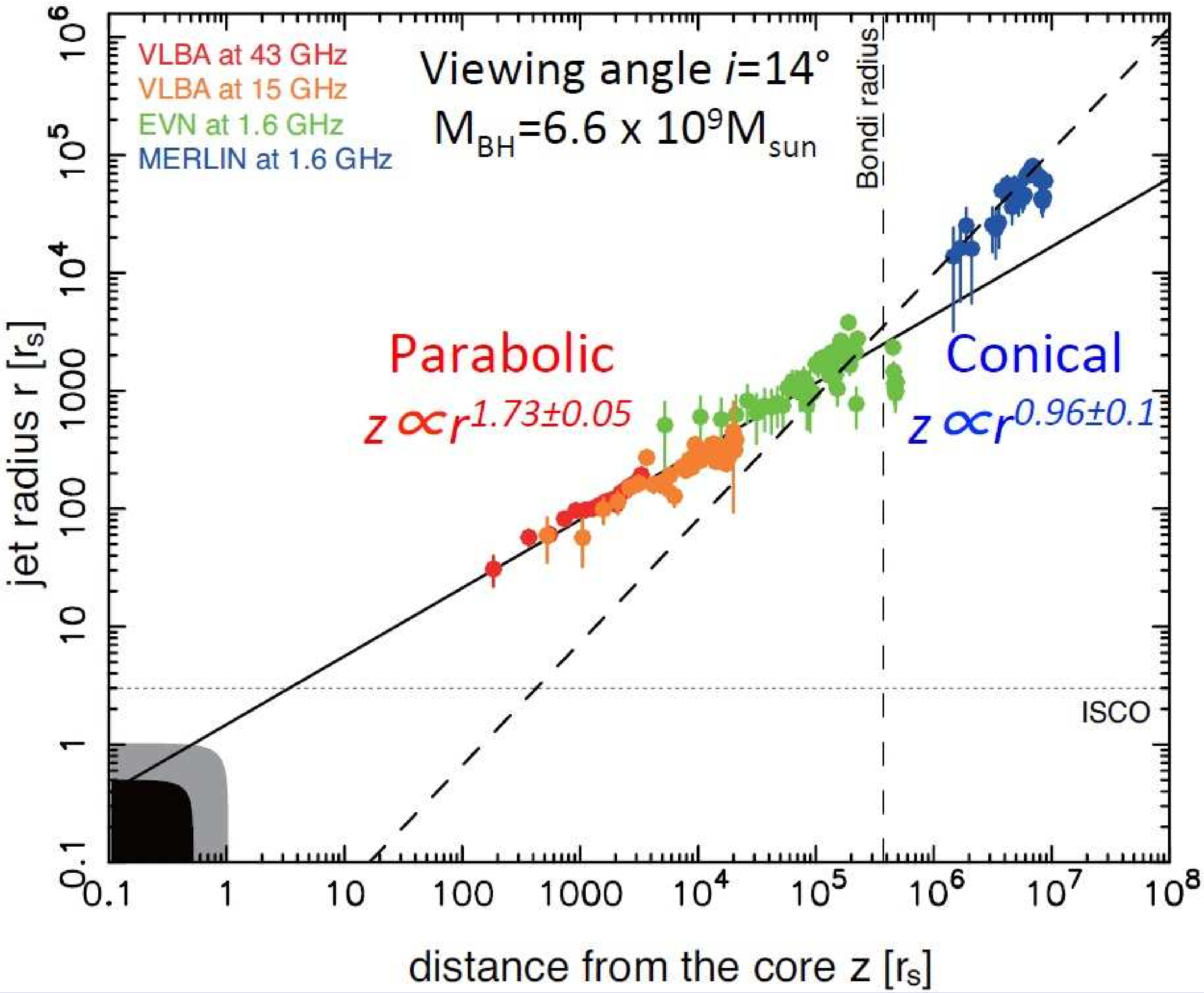

γ_i=50, σ_j=11
x 10^4
6
5
4
3
2
z
0
500
1000
1500
ϖ
γ
100
200
300
400
500
600


600
500
400
300
200
100
γ
2
3
4
5
6
z
x 10^4


0.03
0.025
0.02
0.015
0.01
0.005
0
θ
2
3
4
5
6
z
x 10^4

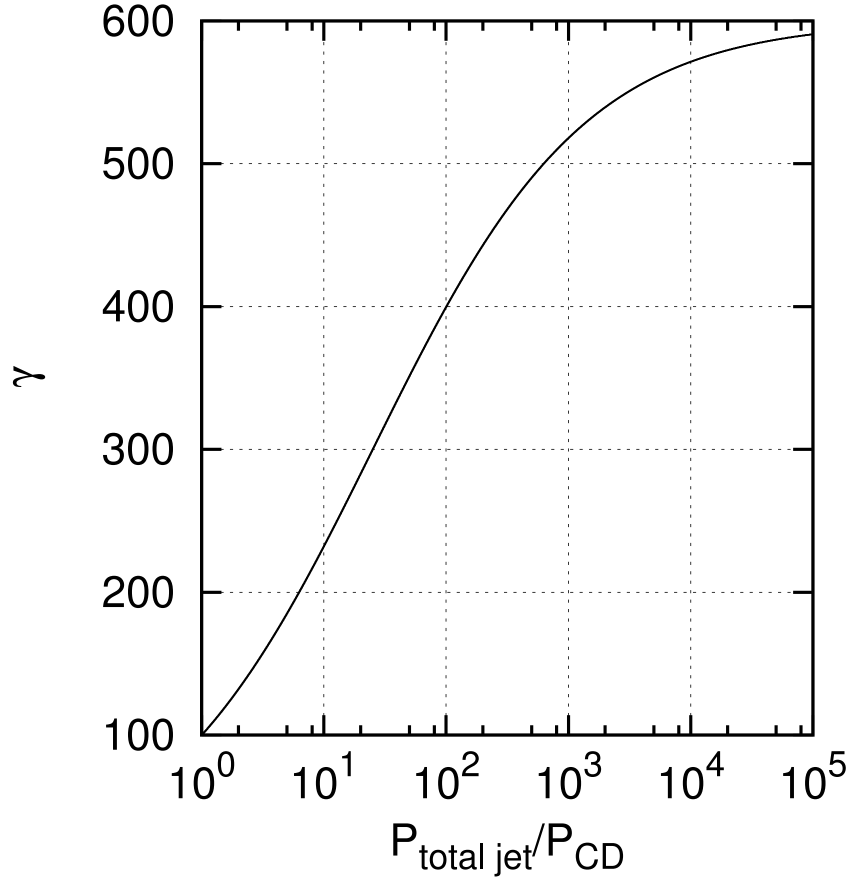
Does it work in AGNs?

(Asada & Nakamura 2011)



The role of the environment

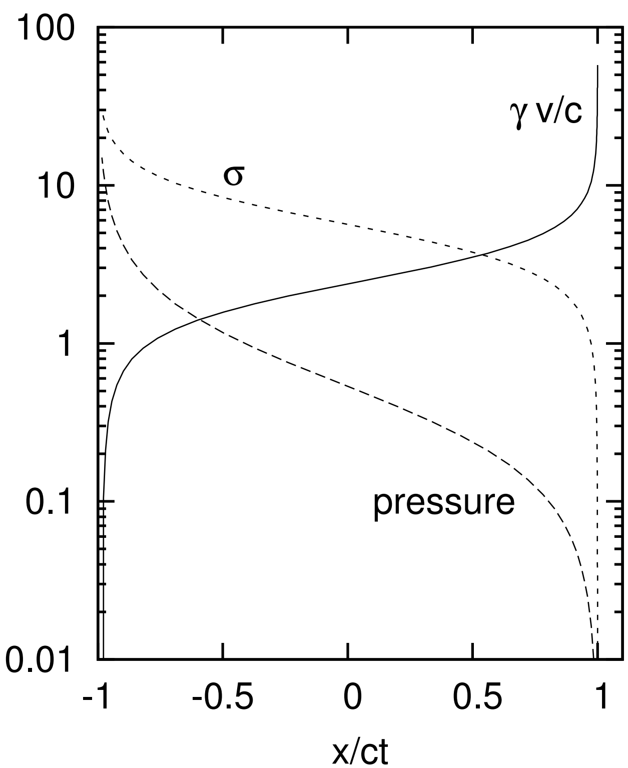
- for nonzero ρ_{ext} Riemann problem: rarefaction on the left state / contact discontinuity / shock on the right



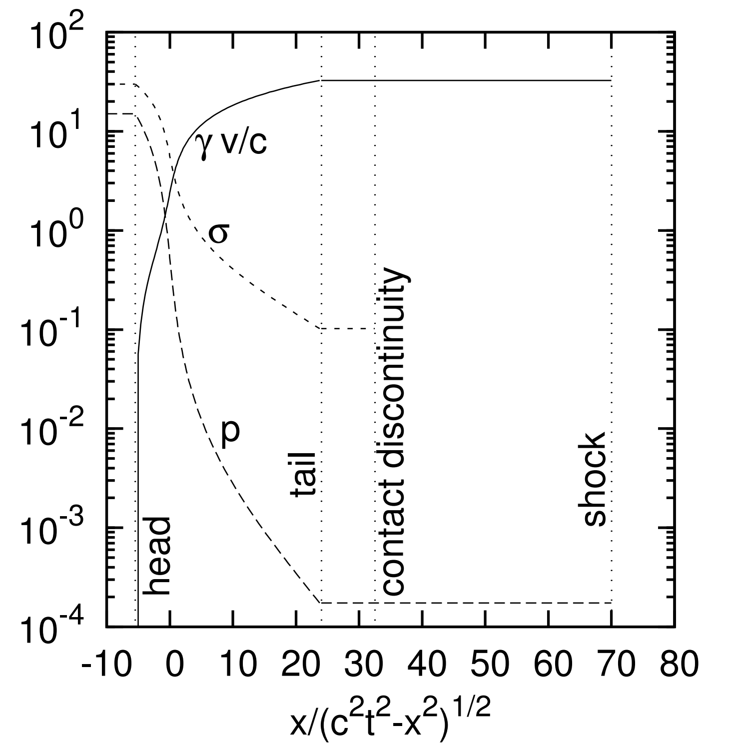
(for $\gamma_j = 100$, $\sigma_j = 1$)

- matching of speed and total pressure at the contact discontinuity gives the solution on the left and right
  (Marti+1994, Lyutikov 2010 for time-dependent problem; Katsoulakos & Vlahakis in preparation for the steady-state)

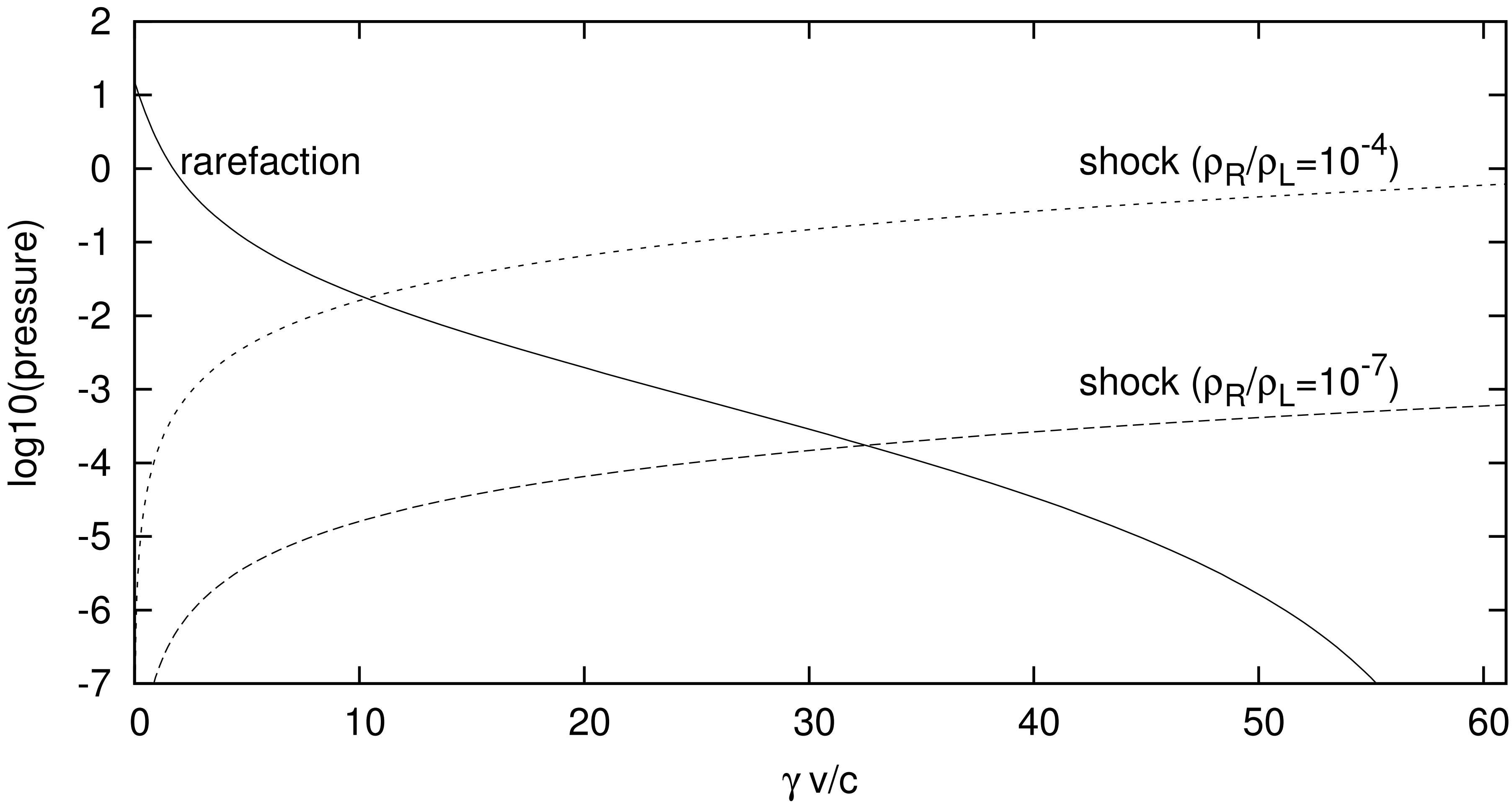
- time-dependent example: impulsive acceleration
  (Granot, Komissarov & Spitkovsky 2011)



for $\rho_R/\rho_L = 0$



for $\rho_R/\rho_L = 10^{-7}$, $P_R = 0$



- in AGNs $\rho_{ext}/\rho_j \gg 1$, so rarefaction unlikely to work
- not clear, see Millas' talk

Summary

⋆ The collimation-acceleration paradigm provides a viable explanation of the dynamics of relativistic jets

⋆ bulk acceleration up to Lorentz factors $\gamma_\infty \gtrsim 0.5\frac{\mathcal{E}}{Mc^2}$
caveat: in ultrarelativistic GRB jets $\vartheta \sim 1/\gamma$

⋆ Rarefaction acceleration

- further increases γ
- makes GRB jets with $\gamma\vartheta \gg 1$
- steady shock creation (?)
- unlikely to work in AGN jets

⋆ The jet-environment interaction is complicated but important to clarify

Acknowledgments

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