Pressure variations in the environment of astrophysical jets induce rarefaction waves propagating inside the jet. Their implications on the jet dynamics will be discussed, focusing on the bulk acceleration and its efficiency in GRB and AGN jets.
Rarefaction waves in magnetized astrophysical jets

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Outline

• “standard” magnetic acceleration (related to collimation)
• rarefaction acceleration
• models – application to GRBs – discussion for AGNs
Magnetized outflows

- Extracted energy per time $\dot{E}$ mainly in the form of Poynting flux (magnetic fields tap the rotational energy of the compact object or disk)
  \[
  \dot{E} = \frac{c}{4\pi} \frac{r}{r_{1c}} B_p B_\phi \times \text{(area)} \approx \frac{c}{2} B^2 r^2
  \]

- Ejected mass per time $\dot{M}$

- The $\mu \equiv \dot{E} / \dot{M} c^2$ gives the maximum possible bulk Lorentz factor of the flow

- Magnetohydrodynamics: matter (velocity, density, pressure) + large scale electromagnetic field
“Standard” model for magnetic acceleration

Component of the momentum equation

\[ \gamma \rho_0 (V \cdot \nabla) (\gamma_w V) = -\nabla p + J^0 E + J \times B \]

along the flow (wind equation): \( \gamma \approx \mu - F \)

where \( F \propto r^2 n \gamma V_p = r^2 \times \text{mass flux} \)

since mass flux \( \times \delta S = \text{const} \),

\( F \propto r^2 / \delta S \propto r / \delta \ell_\perp \)

Acceleration requires the separation between streamlines to increase faster than the cylindrical radius

The collimation-acceleration paradigm:

\( \downarrow \) through stronger collimation of the inner streamlines relative to the outer ones (differential collimation)
transfield component of the momentum equation

\[
\frac{\gamma^2 r}{\mathcal{R}} \approx \left( \frac{2I}{\Omega B_p r^2} \right)^2 r \nabla \ln \gamma - \gamma^2 \frac{r_{1c}^2}{r^2} \nabla r, \text{ with } \nabla \sim \frac{1}{r}, \quad r_{1c} = \frac{c}{\Omega},
\]

simplifies to

\[
\frac{\gamma^2 r}{\mathcal{R}} \approx \underbrace{1}_{\text{inertia}} - \underbrace{\gamma^2 \frac{r_{1c}^2}{r^2}}_{\text{centrifugal}}
\]

- if centrifugal negligible then \( \gamma \approx \frac{z}{r} \) (since \( \mathcal{R}^{-1} \approx -\frac{d^2 r}{d z^2} \approx \frac{r}{z^2} \))

power-law acceleration regime

(for parabolic shapes \( z \propto r^a \), \( \gamma \) is a power of \( r \))

- if inetria negligible then \( \gamma \approx \frac{r}{r_{1c}} \) linear acceleration regime

- if electromagnetic negligible then ballistic regime
Simulations of relativistic jets
Komissarov, Barkov, Vlahakis, & Königl (2007)

Left panel shows density (colour) and magnetic field lines. Right panel shows the Lorentz factor (colour) and the current lines.
\( \gamma \sigma \) (solid line), \( \mu \) (dashed line) and \( \gamma \) (dash-dotted line) along a magnetic field line as a function of cylindrical radius.
Komissarov, Vlahakis, Königl, & Barkov 2009
Caveat: $\gamma \vartheta \sim 1$ (for high $\gamma$)

During the afterglow $\gamma$ decreases
When $1/\gamma > \vartheta$ the observed flux decreases faster with time
• with $\gamma \vartheta \sim 1$ very narrow jets ($\vartheta < 1^\circ$ for $\gamma > 100$) $\rightarrow$ early breaks or no breaks at all

• this is a result of causality (across jet): outer lines need to know that there is space to expand

• equivalent to $R \approx \gamma^2 r$ (transfield force balance)

• Mach cone half-opening $\theta_m$ should be $> \vartheta$

With $\sin \theta_m = \frac{\gamma f c_f}{\gamma V_p} \approx \frac{\sigma^{1/2}}{\gamma}$ the requirement for causality yields $\gamma \vartheta < \sigma^{1/2}$.

For efficient acceleration ($\sigma \sim 1$ or smaller) we always get $\gamma \vartheta \sim 1$
Rarefaction acceleration
Rarefaction acceleration
Rarefaction acceleration
Rarefaction simple waves

At $t = 0$ two uniform states are in contact:

This Riemann problem allows self-similar solutions that depend only on $\xi = x/t$.

• when $\rho_R/\rho_L = 0$ simple rarefaction wave
At $t > 0$:

\[ v_x = \frac{1}{\gamma_j} \frac{2\sigma_j^{1/2}}{1 + \sigma_j} \left[ 1 - \left( \frac{\rho}{\rho_j} \right)^{1/2} \right], \quad \gamma = \frac{\gamma_j (1 + \sigma_j)}{1 + \sigma_j \rho / \rho_j}, \quad \rho = \frac{4\rho_j}{\sigma_j} \sinh^2 \left[ \frac{1}{3} \text{arcsinh} \left( \sigma_j^{1/2} - \frac{\mu_j x}{2} \right) \right] \]

\[ V_{\text{head}} = -\frac{\sigma_j^{1/2}}{\gamma_j}, \quad V_{\text{tail}} = \frac{1}{\gamma_j} \frac{2\sigma_j^{1/2}}{1 + \sigma_j}, \quad \Delta \vartheta = V_{\text{tail}} < 1/\gamma_i \]
The colour image in the Minkowski diagrams represents the distribution of the Lorentz factor and the contours show the worldlines of various fluid parcels. (see also Aloy & Rezzolla 2006 for HD, Mizuno+2008 for MHD)
Simulation results
Komissarov, Vlahakis & Königl 2010
(see also Tchekhovskoy, Narayan & McKinney 2010)
Steady-state rarefaction wave
Sapountzis & Vlahakis (2013)

- “flow around a corner”
- planar geometry
- ignoring $B_p$ (nonzero $B_y$)
- similarity variable $x/z$ (angle $\theta$)
- generalization of the nonrelativistic, hydrodynamic rarefaction (e.g. Landau & Lifshitz)
\[ \theta_{\text{head}} = -\frac{\sigma_j^{1/2}}{\gamma_j} \]

\[ \theta_{\text{tail}} = \frac{2\sigma_j^{1/2}}{\gamma_j(1+\sigma_j)} \]

\[ \sigma = \left( \sigma_j \gamma_j x_i / z \right)^{2/3} \]

\[ \sigma = 1 \text{ at } r = \sigma_j \gamma_j |x_i| = 7 \times 10^{11} \sigma_j \left( \frac{|x_i|}{R_*/\gamma_j} \right) \left( \frac{R_*}{10R_\odot} \right) \text{ cm} \]

Time-dependent (left) and steady-state (right) rarefaction (similar; \( ct \rightarrow z \))

(distance unit = \( R_*/\gamma_j \sim 10^{10} \) cm)
**Axisymmetric model**

Solve steady-state axisymmetric MHD eqs using the method of characteristics (Sapountzis & Vlahakis in preparation)
Reflection of the wave from the axis

\[ \gamma_j = 100, \ \sigma_j = 1, \ \rho_{ext} = 0 \]
Reflection causes sudden deceleration – standing shock (?)
Does it work in AGNs?  
(Asada & Nakamura 2011)

Viewing angle $i=14^\circ$

$M_{BH}=6.6 \times 10^9 M_{\odot}$

Parabolic $z \propto r^{1.73\pm0.05}$

Conical $z \propto r^{0.96\pm0.1}$

Accretion and Outflows throughout the scales  
2 October 2014, Lyon
The role of the environment

• for nonzero $\rho_{ext}$ Riemann problem: rarefaction on the left state / contact discontinuity / shock on the right

\[
\gamma \quad \text{against} \quad \frac{P_{\text{total jet}}}{P_{\text{CD}}}
\]

(for $\gamma_j = 100$, $\sigma_j = 1$)
• matching of speed and total pressure at the contact discontinuity gives the solution on the left and right (Marti+1994, Lyutikov 2010 for time-dependent problem; Katsoulakos & Vlahakis in preparation for the steady-state)

• time-dependent example: impulsive acceleration (Granot, Komissarov & Spitkovsky 2011)

\[ \gamma v/c \]
\[ \sigma \]
\[ \text{pressure} \]

\[ x/ct \]

for \( \rho_R/\rho_L = 0 \)

\[ 100 \]
\[ 10 \]
\[ 1 \]
\[ 0.1 \]
\[ 0.01 \]

\[ x/(c t^2 - x^2)^{1/2} \]

\[ \gamma v/c \]
\[ \sigma \]
\[ p \]

for \( \rho_R/\rho_L = 10^{-7}, P_R = 0 \)

Accretion and Outflows throughout the scales

2 October 2014, Lyon
• in AGNs $\rho_{ext}/\rho_j \gg 1$, so rarefaction unlikely to work

• not clear, see Millas’ talk
Summary

★ The collimation-acceleration paradigm provides a viable explanation of the dynamics of relativistic jets

★ bulk acceleration up to Lorentz factors $\gamma_\infty \gtrsim 0.5 \frac{E}{Mc^2}$
   caveat: in ultrarelativistic GRB jets $\vartheta \sim 1/\gamma$

★ Rarefaction acceleration
   ● further increases $\gamma$
   ● makes GRB jets with $\gamma \vartheta \gg 1$
   ● steady shock creation (?)
   ● unlikely to work in AGN jets

★ The jet-environment interaction is complicated but important to clarify
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